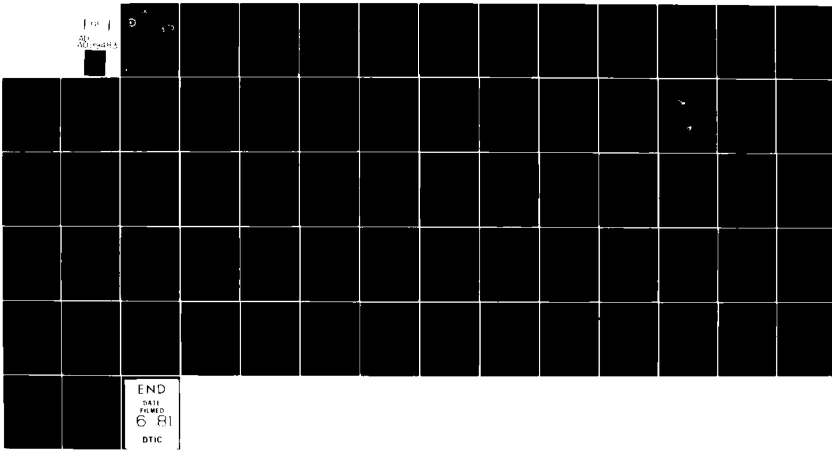
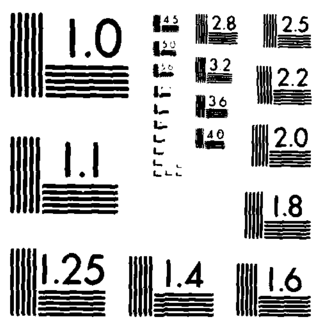
AD-A099 483 DEFENSE COMMUNICATIONS ENGINEERING CENTER RESTON VA F/G 17/2
ON THE PLACEMENT AND SIZING OF CONFERENCE DIRECTORS IN THE CONU--ETC(U)
NOV 80 M J FISCHER, G W SWINSKY
UNCLASSIFIED DCEC-TN-15-80 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

LEVEL II

12

TN 15-80

DEFENSE COMMUNICATIONS ENGINEERING CENTER

TECHNICAL NOTE NO. 15-80

ON THE PLACEMENT AND SIZING OF CONFERENCE DIRECTORS IN THE CONUS AUTOVON

NOVEMBER 1980

407519

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

UNCLASSIFIED November 1980

407 519

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
(14) DCEC-TN-15-80

2. GOVT ACCESSION NO.
AD-A099 483

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
(6) ON THE PLACEMENT AND SIZING OF CONFERENCE DIRECTORS IN THE CONUS AUTOVON,

5. TYPE OF REPORT & PERIOD COVERED
(9) Technical Note

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
(10) M. J. Fischer
G. W. Swinsky

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Defense Communications Engineering Center
Systems Engineering Division, R700
1860 Wiehle Avenue, Reston, VA 22090

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
N/A (12) 71

11. CONTROLLING OFFICE NAME AND ADDRESS
Same as 9

12. REPORT DATE
(11) Nov[redacted]80

13. NUMBER OF PAGES
71

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
N/A

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
N/A

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release, distribution unlimited.

MAY 29 1981

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
N/A

18. SUPPLEMENTARY NOTES
Review relevance 5 years from submission date.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Conference Directors
CONUS AUTOVON
Network Optimization
Queueing

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this technical note we report on the study that was conducted in answering the following questions:

- How many Conference Directors should there be in CONUS AUTOVON and where should they be located?
- What is the required port sizing to meet the conference traffic requirements?

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

407519

- What is the impact of accommodating the conferencing traffic requirements on CONUS AUTOVON?

The analytic and computer methods used to answer these questions, as well as the study results, are discussed in the Technical Note.

TECHNICAL NOTE NO. 15-80

ON THE PLACEMENT AND SIZING OF CONFERENCE DIRECTORS IN THE CONUS AUTOVON

NOVEMBER 1980

Prepared by:

- M. J. Fischer
- G. W. Swinsky

Approved for Publication:

G. E. LaVEAN
Chief, Systems Engineering Division

FOREWORD

The Defense Communications Engineering Center (DCEC) Technical Notes (TN's) are published to inform interested members of the defense community regarding technical activities of the Center, completed and in progress. They are intended to stimulate thinking and encourage information exchange; but they do not represent an approved position or policy of DCEC, and should not be used as authoritative guidance for related planning and/or future action.

Comments or technical inquiries concerning this document are welcome, and should be directed to:

Director
Defense Communications Engineering Center
1860 Wiehle Avenue
Reston, VA 22090

TABLE OF CONTENTS

		Page
	EXECUTIVE SUMMARY	vi
I.	INTRODUCTION	1
II.	STUDY PROCEDURES AND METHODOLOGY	3
III.	SYSTEMS ANALYSIS	14
IV.	SIGNIFICANT FINDINGS AND CONCLUSIONS	29
	REFERENCES	30
APPENDIXES		
A	GENERATION OF SVIP CONUS CONFERENCING TRAFFIC REQUIREMENTS	A-1
B	CONFERENCE DIRECTOR PERFORMANCE AND SIZING MODEL	B-1

LIST OF ILLUSTRATIONS

Figure	Title	Page
1	BASIC STUDY STRUCTURE	4
2	PROBABILITY DISTRIBUTION OF NUMBER OF CONFEREES IN A CONFERENCE	6
3	CONUS AUTOVON CD CONFIGURATION	8
4	ROUTING FOR A 10 CONFEREE CONFERENCE	11
5	SELECTION OF TWO CONUS/CANADA SWITCHING SITES FOR CD PLACEMENT	16
6	SELECTION OF TEN CONUS/CANADA SWITCHING SITES FOR CD PLACEMENT	17
7	IST MILEAGE FOR THREE GENERATIONS OF A 48 CONFERENCE REQUIREMENTS SET	18
8	IST MILEAGE VS. NUMBER OF CD'S (48 CONFERENCES)	20
9	TRANSACTION ACTIVITY FOR AN AVERAGE CD	22
10	PORT REQUEST ACTIVITY FOR AN AVERAGE CD	23
11	NUMBER OF CD'S UTILIZED BY A TYPICAL CONFERENCE	24
12	PORTS REQUESTED BY AN AVERAGE CONFERENCE	25
13	TOTAL NUMBER OF PORTS IN SYSTEM (P10 BLOCKING ON CD)	28
A-1	LISTING OF SVIP LOCATIONS	A-2
A-2	DISTRIBUTION OF NUMBER OF ORIGINATION CONFERENCES BY LOCATION	A-6
A-3	LISTING OF CONFERENCE REQUIREMENTS	A-9
B-1	CYCLIC BEHAVIOR OF LOSS PROBABILITIES	B-15
B-2	SENSITIVITY TO VARIANCE OF OFFERED LOAD	B-16
B-3	BEHAVIOR OF SYSTEM WHEN $Q_1=0$	B-19
B-4	COVARIANCE OF REQUEST FOR PORTS	B-20
B-5	PROBABILITY DISTRIBUTION OF THE NUMBER OF CALLS IN SYSTEM BY CLASS	B-21

LIST OF TABLES

Table	Title	Page
I.	SELECTION OF CD LOCATIONS FOR VARIABLE NUMBER OF CD EQUIPMENTS	15
II.	COST OF AUTOVON TRUNKS TO SUPPORT THE ADDITIONAL CONFERENCING TRAFFIC	19
III.	DISTRIBUTION OF CD PORTS FOR VARIABLE NUMBER OF CD'S IN THE NETWORK (P.10 BLOCKING)	27

EXECUTIVE SUMMARY

In this technical note we report on the study that was conducted in answering the following questions:

- How many Conference Directors should there be in CONUS AUTOVON and where should they be located?

- What is the required port sizing to meet the conference traffic requirements?

- What is the impact of accommodating the conferencing traffic requirements on CONUS AUTOVON?

The analytic and computer methods used to answer these questions are discussed in the Technical Note. One general conclusion of our study is that the number and location of the Conference Directors is highly dependent on the traffic requirements. Another result is that the optimal number of Conference Directors for the traffic requirements used in our study is somewhere between 2 and 4 Conference Directors.

I. INTRODUCTION

In references [1] and [2] the Defense Communications Engineering Center (DCEC) was directed to study the effects on today's CONUS AUTOVON network brought about by overlaying a common user Secure Voice Conferencing capability (SVIP), which makes use of Conference Director (CD) processor controlled equipments placed at present AUTOVON switching sites. DCEC was directed to look at many issues of this problem, but four basic questions were posed to Branch R720 of the Systems Engineering Division. These questions are:

1. How many Conference Directors should there be in CONUS AUTOVON?

2. Where should they be located?

3. What is the port sizing required to meet the conference traffic requirements?

4. What is the impact of accommodating the conferencing traffic requirements on CONUS AUTOVON?

In this technical note we discuss the methods used and study results obtained in answering these questions.

In order to address these questions from an analytic and quantifiable approach several sets of data had to be obtained. The major one is a specification of the conferencing traffic requirements to be used in the study. By this we mean the point-to-point offered conference erlang loads, and the structural makeup of a particular conference. For each conference, one has to know the number and the locations of all conferees associated with the particular conference. Without such information no quantifiable study can be made. Furthermore, these conferencing traffic requirements are the biggest driver in the placement of a CD within CONUS. That is, one is not going to place a CD on the west coast of the United States if all the traffic requirements for a conference are on the east coast.

We were unable to find any such set of traffic requirements that was based on actual measured traffic, such as contained in the Traffic Data Collection System (TDCS) of AUTOVON. Therefore, we were forced to generate our own conference traffic. The method that was used to generate this traffic is presented in Appendix A. It was coordinated within DCEC for comments and was agreeable to all interested parties.

Another major piece of work that had to be accomplished dealt with answering question 3., CD port sizing. The normal method of sizing ports or trunks is to determine the offered load trying to use the ports, select the appropriate queueing model and interactively increase the number of ports until the desired measure of performance is met. The problem which arises in the context of conference directors is that no such queueing model exists. A conference call is similar to a two party call except that, rather than requiring one port, it can require two or more ports depending on the nature of the particular conference. Thus, the standard Erlang Loss System [3] equations cannot be used. However, we have developed a queueing model (see Appendix B) that predicts the performance of conferences requesting use of the ports on the CD's.

These two problems posed the major developmental efforts in the study. The remaining portion of the study was accomplished by a simple straightforward application of several of the Network Design and Analysis tools developed by R720 for other efforts within DCEC. Section II of this technical note discusses procedures and methodologies used in this study. The study results are given in section III along with the graphs and tables that were used to generate these results. Finally, section IV contains significant findings and conclusions.

II. STUDY PROCEDURES AND METHODOLOGY

The basic flow of the study is shown in Figure 1. From a given set of possible Conference Director (CD) locations and for a particular number of CD's (say k), the optimal k CD locations were found. The conference traffic subscribers were then homed and their traffic distributed within CONUS via the appropriate routing. For each CD, the traffic trying to use the ports was then collected and used to size the ports for the desired grade of service. Finally, the effect in terms of additional trunking cost to support the conference traffic in CONUS AUTOVON was computed. The value of k was increased and the procedure was continued. In the study, we varied k from 2 to 20 in increments of 2. This section describes what was basically done in each of these steps and the assumptions that were made.

Before any study of network behavior can begin, a detailed set of conferencing traffic requirements (point to point offered traffic) for the SVIP user community must be known. This set of requirements must be sufficiently detailed to indicate the geographic point to point (or in the case of conferencing, point to points) flows of conference voice traffic during a typical busy hour. The major assumptions used in generating these requirements were:

- The SVIP CONUS user community is located at sites already having access to the AUTOVON network and this existing access will be used in establishing the conferencing.
- The AUTOVON network in conjunction with processor controlled conference directors placed at switching sites will be responsible for the establishment of the connectivity required by a given conference.
- The set of conference requirements, indicating locations of originators and conferees for each conference, is an accurate representation of the steady state, day to day peacetime oriented demand for SVIP conferencing in CONUS during a typical busy hour.

No such set of detailed requirements exist which satisfies the above assumptions. In CONUS AUTOVON today, conferencing is conducted in a number of distinct ways. One example is that of a specilized command, such as NORAD, which implements conferencing specifically tailored to its own unique type of mission. Many of the conferences are prearranged and established, conditioned on the occurrence of an event rather than on a purely random basis. Another example is the 4-wire subscriber who is able to initiate random and prearranged conferences via two AUTOVON special assist operators at the Monrovia and San Luis Obispo switching centers, where traffic statistics of these conferences are generally not taken on a regular basis. A third example is that of a conference initiated by an authorized user who calls to a PBX and has a special attendant operator establish a conference by sequentially dialing in the conferees and manually connecting them to a bridge. In all of these examples, no complete and consistent set of requirements sufficient to perform network analyses exists.

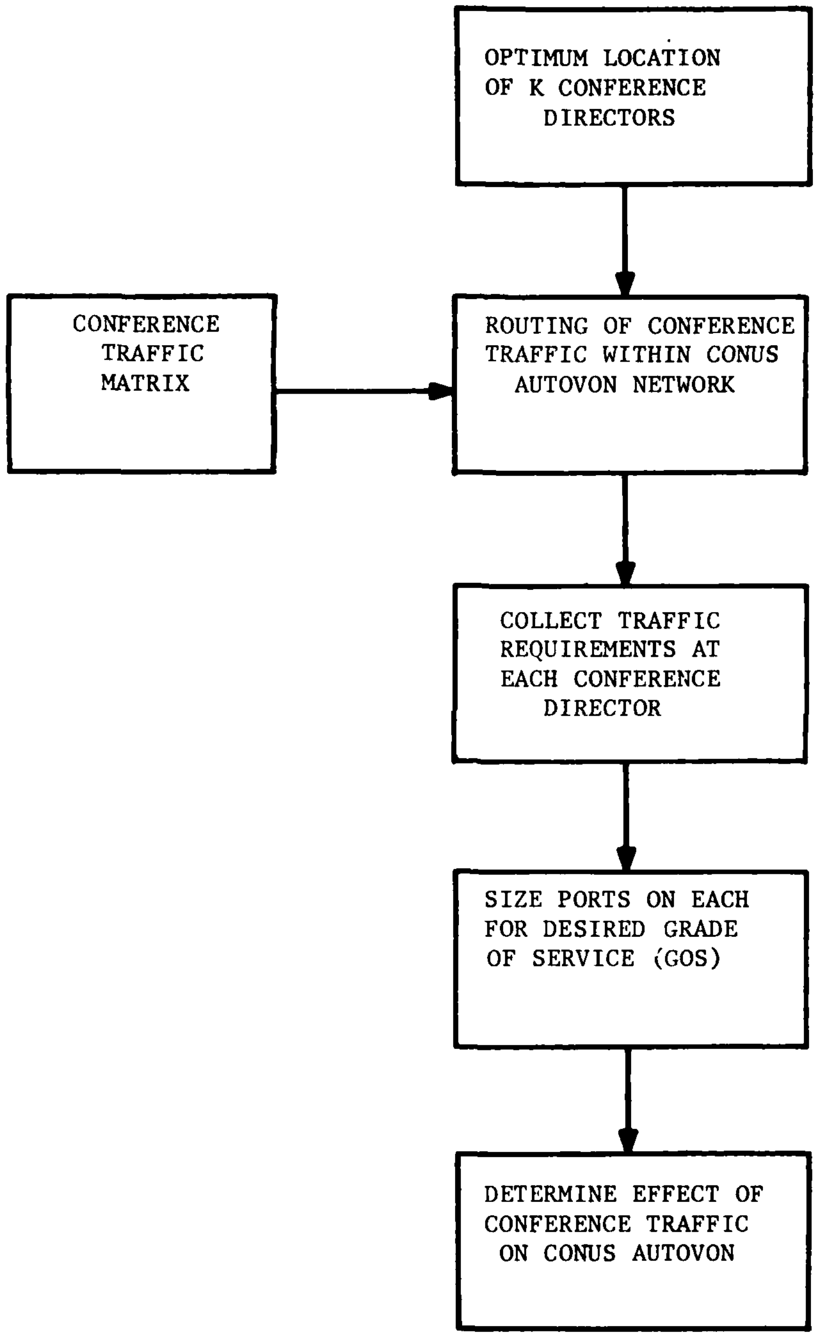


Figure 1. Basic Study Structure

In order to examine network behavior during predicted common user SVIP conferencing in the CONUS, conferencing traffic was computer generated, using simulation techniques based on an existing data base of SVIP CONUS locations and the number of busy hour originating two party SVIP calls emanating from these locations. For a detailed discussion of the procedures used, see Appendix A. This procedure utilized the following major assumptions:

- The number of individual common user busy hour SVIP CONUS conferences originating from each of the 251 SVIP user locations in the above data base is proportional to the amount of two party SVIP traffic emanating from these locations.

- For each such conference, the number of conferees in the conference follows a preset distribution.

- The location of each user conferee was randomly chosen from locations in the data base, but with a probability proportional to the amount of SVIP two party erlang traffic; i.e., the locations generating higher amounts of two party SVIP traffic are more likely to be picked as conferee locations.

Two sets of traffic requirements were generated, a low traffic case of 48 conferences per busy hour and a high of 210. A set of 48 conferences, chosen as above, was generated using Monte Carlo sampling via a random number generator. So as not to bias the resulting analysis with a particular random number seed, two additional sets of conference requirements each with 48 conferences were also generated, using the same rules as above; all three sets of conference requirements were individually overlaid on the CONUS AUTOVON and subjected to the analyses which follow. Each set has the same number of originating conferences at the same locations, but the number and locations of the conferees of each conference varies according to the different samples from the underlying distributions used. In this manner, the sensitivity of the major results to the random sampling process can be examined. The baseline set of 48 conferences per busy hour was generated using a procedure which assumed that of the first 36 SVIP locations which generate the greatest amounts of SVIP two-party voice communications, the highest third of those would originate two busy hour conferences and the other two-thirds would generate one busy hour conference. The remaining 215 SVIP locations were assumed to originate no busy hour conferences. The assumed distribution of number of conferees in a given conference is shown in Figure 2. The theoretical mean of this distribution is 5.65 conferees per conference, and the mode of the distribution occurs at 4 to 5 conferees per conference. The actual number of conferees over the 48 conferences was 253, 297 and 282 for the three independent generations, which represents a sample average of 277.3 overall or about 5.78 per conference. These numbers are in close agreement with the theoretical expected value. For the 210 conferences per busy hour, the destination distribution of the conferees was the same as the 48 conference case.

In order to get a perspective on the comparative amounts of traffic involved, the following is illustrative of the quantities of interest. The total two-party originating erlang load offered to the network by the 251

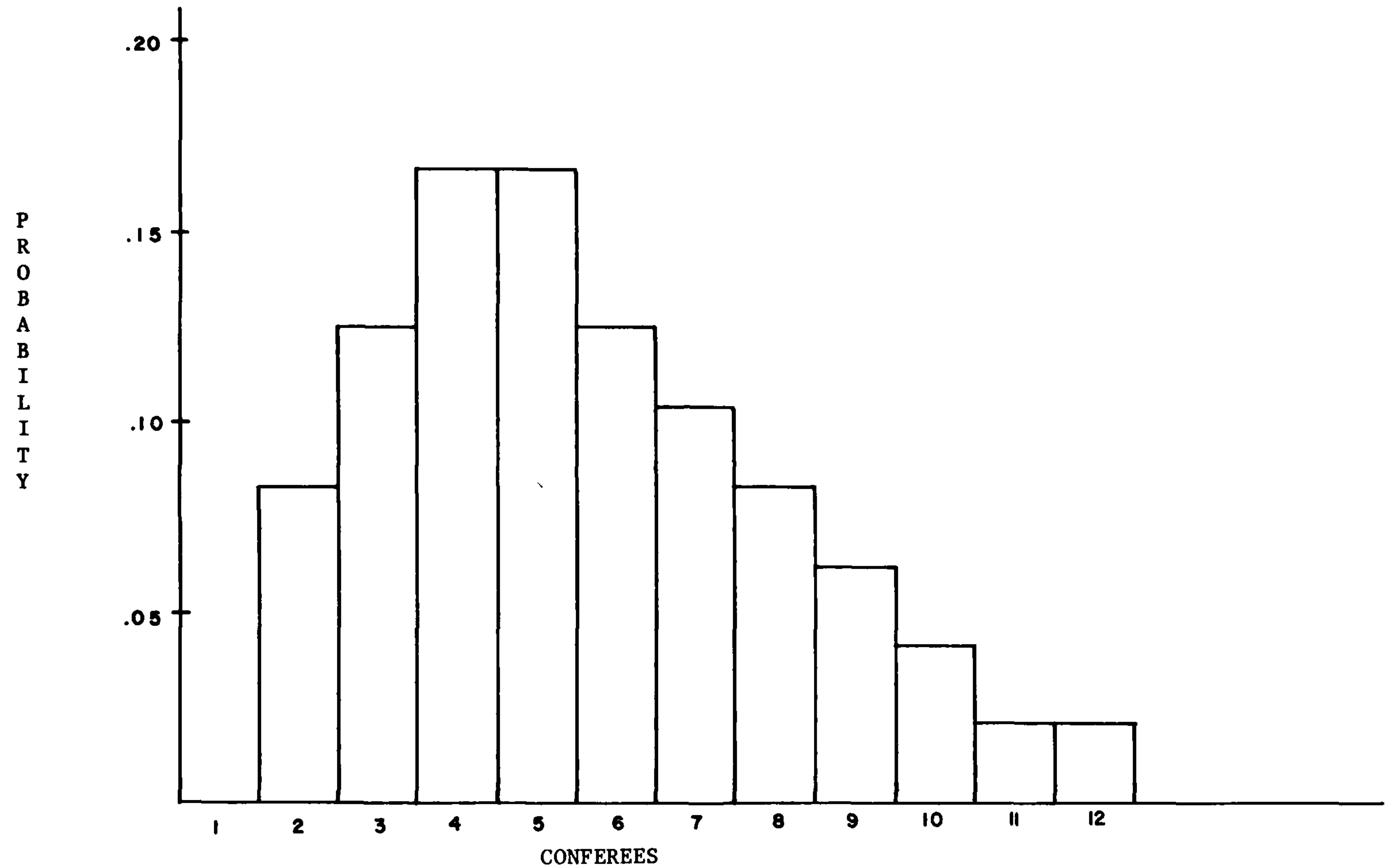


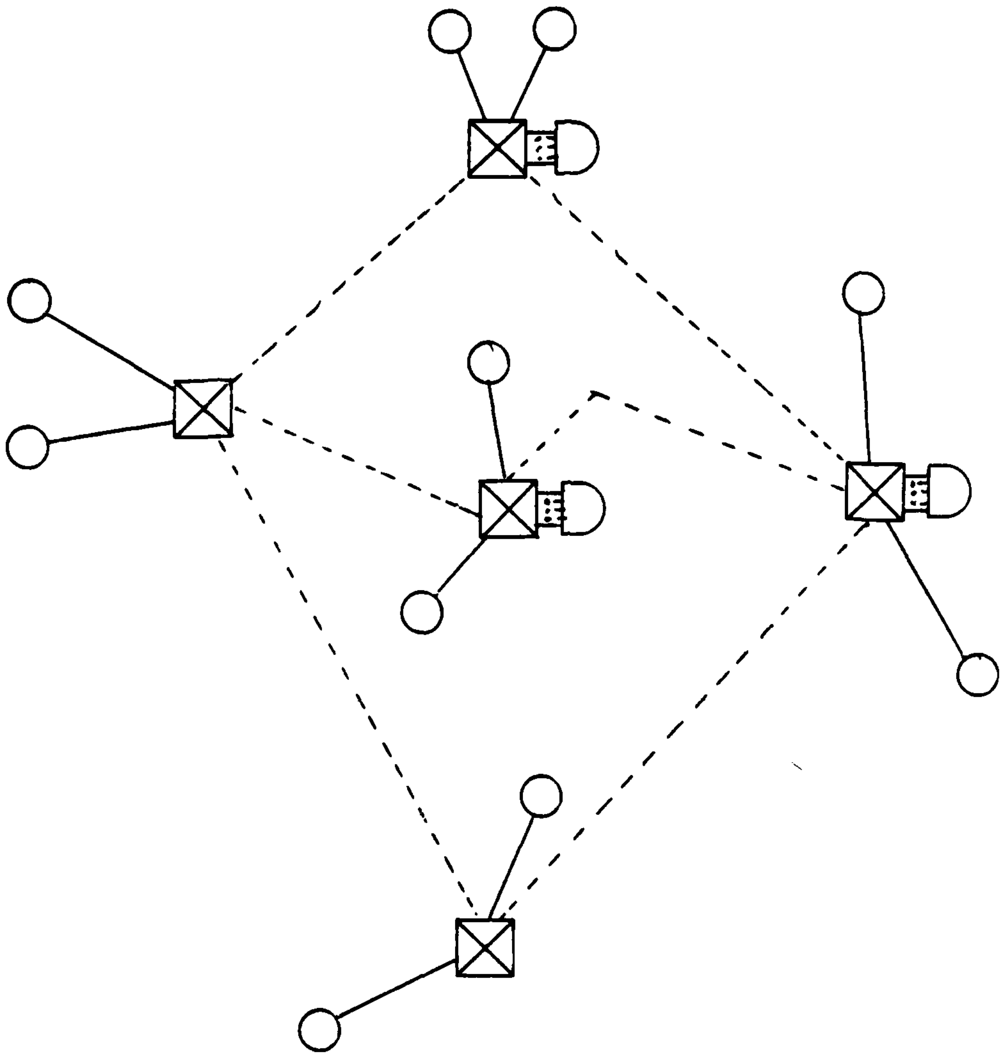
Figure 2. Probability Distribution of Number of Conferees in a Conference

SVIP subscriber locations was approximately 545 busy hour erlangs. In comparison, the clear voice network offered load to the CONUS AUTOVON network is presently running 5157 erlangs, which represents a traffic load averaged over the two busy hours of the day (one in the morning, one in the afternoon) for the normally busy month of January 1980.

Throughout this study, an average holding time of 10 minutes was assumed for a typical conference. The amount of originating conference erlangs for the 48 conferences would then be 8 (=48x10/60) erlangs, but this figure only represents the originations and does not take into account the number of conferees and their locations in the conference. Since we have 5.65 average conferees (not including originator) in a conference, a rough estimate of the offered load to the network is 8x5.65=45.2 erlangs (about 8.5% of the SVIP two party load). For the 210 case, we have 185.9 erlangs offered to the network, or 37.2% of the SVIP two party load. These figures roughly correspond to what would be expected had each originator placed independent calls to each of his conferees. In actuality, these calls are not placed independently but routed in the network via a minimum spanning tree between involved CD's. This routing is in turn dependent on both the specific network CD configuration under consideration and the source-destination characteristics of the particular conference. All of these factors will be considered in the following analyses, and actual switch-to-switch offered erlangs resulting from the actual flow of conferences in today's AUTOVON will be discussed. The study was conducted using the traffic matrix which resulted from the 48 conferences/busy hour and also from the 210 conferences/busy hour.

The first step in a particular run of the study was to fix the number of CD's, say k (the values of k that were considered were k=2,4,...,20); the next step was to determine the optimal location of k CD's from the candidate list of possible CD locations. We assumed that the list of possible CD locations considered the current CONUS AUTOVON switching sites. Furthermore, we assumed that if a CD was placed at one of these sites it was collocated with the site and would function as just another PBX subscriber to that switch, in terms of obtaining access through the network, either to another CD, or to a conferee in the conference. Access from the CD to its collocated switch is through ports connecting them. The number of such ports will be determined by the actual routing of the conference requirements and subsequent sizing analysis. This study considers CD equipments as common user in nature, and available to and from the AUTOVON network through the collocated switch. It does not consider private access from user locations, although this could be implemented as requirements warrant. Figure 3 illustrates the assumed architecture for this study. The user access lines to AUTOVON switches and the interswitch trunks are those in existence today (May 1980). A SVIP conference originator (user) would access first the switch to which he is homed and then, if no CD were present would be automatically routed to the closest switch which has a CD. The routing of conferences thereafter is discussed in detail later.

Switching locations and not subscriber's locations are chosen in this study as candidate sites for CD placement for a number of reasons. Primarily, the switching locations have already been selected to provide relatively short distance access to the greater number of subscribers. Further, the CD is



LEGEND

Present AUTOVON Switch

CD Collocated with AUTOVON Switch

SVIP User PBX Location

Existing AUTOVON Access

Existing AUTOVON IST

Figure 3. CONUS AUTOVON CD Configuration

considered as a common user equipment eligible for use, for example, by conference originators and conferees at diverse locations, and by other CD's in the routing of a conference. To place a common user CD at a subscriber location would place significant stress on the access line group at that subscriber, and would simultaneously create inefficient access by other nearby subscribers wishing to use the CD.

As discussed earlier, the issue of how many AUTOVON switching sites to select for CD placement is addressed in this study by treating this figure parametrically. That is, an optimum set of two best sites was chosen and the identity of the switches saved. This analysis was repeated, choosing the best 4 and again choosing the best 6 and so on up to 20 sites, resulting in 10 configurations of varying quantities of switching locations selected. All of the selections were chosen from the full set of 54 CONUS/CANADA AUTOVON switch locations with the goal of optimally reducing the user access to the CD's, as measured by traffic weighted mileage from a user location to the closest switch in the network having a CD, summed up over all user locations. Had firm conferencing traffic statistics been available, these would have been used as weights. However, as explained above, the conferencing traffic itself was generated from two party SVIP traffic originating from the SVIP user locations. As a result, these latter traffic figures were used as weights for the SVIP locations because of their role in determining conferencing traffic and because of their high correlation to the data base of 10,000 SVIP instruments.

We have a computer algorithm which was based on some work originally done at Bell Labs [4, 5]. This algorithm solves the optimal placement problem of the CD's. In general it solves the following problem. Suppose there are 'M' subscriber locations and 'N' candidate locations for CD's of which the optimal 'k' are to be determined. In this application M = 251, corresponding to the SVIP CONUS/CANADA subscriber locations. The value for N is 54, which is the number of CONUS/CANADA switch locations. The value for k was taken to range from 2 to 20 in multiples of 2. We briefly describe how the algorithm works in the following paragraphs. For more details see [4] and [5].

The algorithm is begun by constructing a penalty matrix $P = (p_{i,j})$, where $p_{i,j}$ is the penalty associated with homing subscriber i to a CD at switch j. Out of the N possible CD locations, k are chosen randomly as the initial best k locations. Each subscriber is homed to the nearest of these k locations, and the total penalty for all subscribers is computed using the matrix P. Then the algorithm proceeds by iterative optimal swapping of one location in the current set of best k locations with one location which is not so as to always keep k, the number of chosen locations, fixed. The procedure terminates when and only when the total penalty of the homing cannot be further reduced.

In our application, the penalty function used is a weighted mileage

$$p_{ij} = d_{ij}\, t_i$$

where d_{ij} is the distance in miles from subscriber location i to the j^{th} AUTOVON switch site, and t_i is the amount of busy hour SVIP two party clear voice traffic emanating from location i. The use of traffic as a weight allows for discrimination of the heavier users. The penalty function will tend to produce near optimal selection of candidate CD locations close to the traffic-weighted center of mass of the user locations, because every subscriber must be homed to its closest CD.

Once a specific set of conferences is known, and once a specific network configuration is specified (that is, the placement of a specific number of CD's at specific switch locations), then the process of routing the conference originator traffic to its conferees utilizing the resources of AUTOVON can begin. There are three major stages of this process. First, the originator must access his closest CD. This is assumed to be accomplished by the dialing of a special number which identifies to the subscriber's homed switch a request for the origination of a conference. In the case where a CD is not present at this switch, the call is routed, just as with any other AUTOVON call, to the closest switch which has a CD, and this CD becomes the originating CD for the conference.

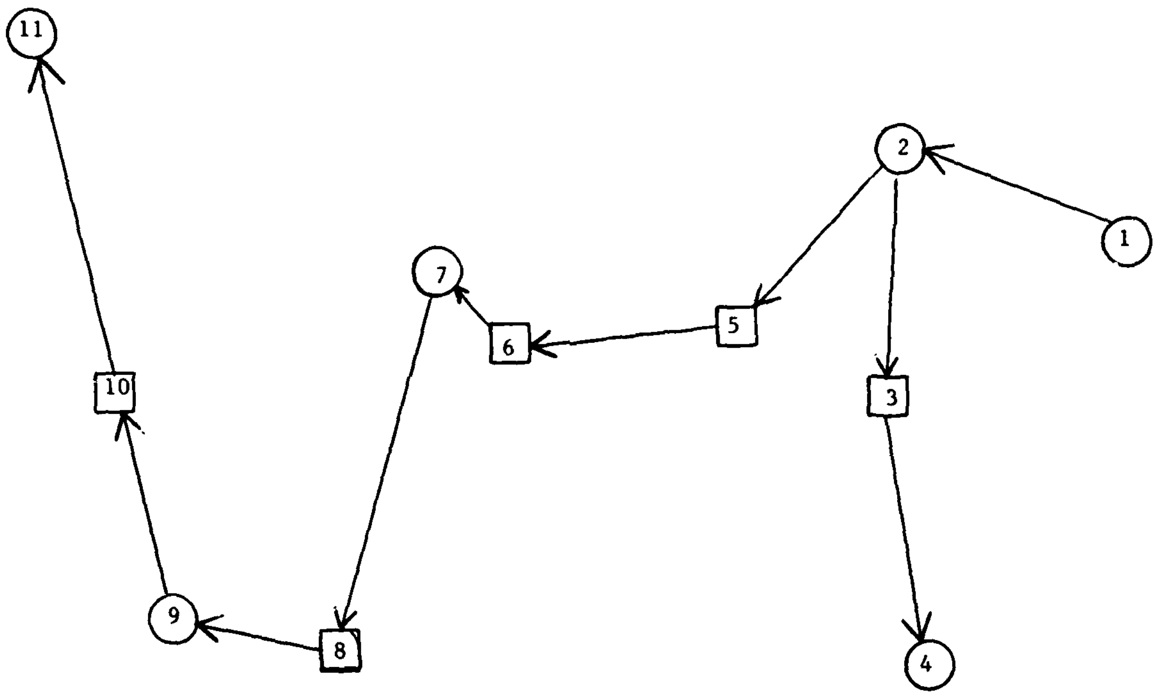
Once the originating CD has been reached, the called numbers of the conferees are made available to it, either through signalling from the user or by table look-up at the CD (prearranged conference). At this point, there are a number of possible ways for the originating CD to establish connectivity with the conferees; for instance, he could place individual calls to each of them.

This CD to CD call routing may not be optimal; for example, consider an originator homed on Lodi, California and conferees homed on Littleton, Massachusetts, Mosely, Virginia and Polk City, Florida. Placing three calls utilizing long haul trunking facilities could be very expensive in terms of AUTOVON resources. A better solution would be to first tandem one call to an East Coast CD who would then proceed to make three shorter distance calls to the conferees. This would involve lesser use of Interswitch Trunk (IST) route mileage on the AUTOVON backbone, and usage of more CD's and a fewer number of ports per CD per typical conference. In addition, there would be a slightly higher number of calls placed in the backbone, but the route mileage of these calls would be lower overall. In general, there is a trade-off between the AUTOVON IST usage and the number of CD's and corresponding ports used in the system. A major assumption of this study is to favor implementation of the type of conference routing which will minimize impact upon AUTOVON resources (IST). To the greatest degree possible then, we routed traffic that made the most use of local tandem CD's wherever possible, to reduce AUTOVON IST route mileage. We determined this CD to CD routing by using a graph theoretic algorithm known as the minimum spanning tree. See [6] for a simplified discussion of this algorithm. The algorithm is a connection hierarchy or tree which connects all CD's involved in a single conference so as to guarantee minimal interconnecting route mileage. The branches in the tree connect CD's at AUTOVON switches, and this connection is effected by placing a call over the shortest path between the switches using the present AUTOVON IST connectivity. This is the second stage of the routing process.

The third and final stage in routing is the reverse of the first, i.e., connecting a tandem (or originating) CD to a conferee by placing an AUTOVON call. When the conferee is not homed to the tandem CD's switch, the call is

Switch Identity

1. Arlington, VA
2. Toledo Junction, OH
3. Williamstown, KY
4. Rockdale, GA
5. Hillsboro, MO
6. Lamar, CO
7. Cheyenne, Mountain, CO
8. Socorro, NM
9. Julian, CA
10. Topaz Lake, NV
11. North Bend, WA



LEGEND

CD Utilized

CD not present, or
not utilized

Figure 4. Routing for a 10 Conferee Conference

routed through the backbone to the conferee's homed switch. Each individual conference in the traffic matrix is routed in this manner. Ports on a CD are seized in each of the following instances:

- o An incoming request (call from a conference originator)
- o An outgoing request (call to another CD)
- o A call to each conferee which has this CD as its local CD.

A detailed example of this routing is now described (see Figure 4). A user homed on the Arlington switch wishes to conduct a conference with 10 conferees homed on the following switches:

Switch	Number of Conferees
Arlington, VA	3
Toledo Junction, OH	2
Cheyenne Mtn, CO	1
Rockdale, GA	1
Julian, CA	2
North Bend, WA	1

We assume each of the above switches has a CD collocated with it. The routing for this conference is as follows: The Arlington CD receives the originator's call, places a call to each of the three conferees homed to Arlington and relays a call to Toledo Junction, accounting for a request of five ports at the Arlington CD (one for the incoming call, one for each of the local conferees and one for the call to Toledo Junction). The Toledo Junction CD receives the call from Arlington, ties in its two local conferees and relays calls to Cheyenne Mountain (via Hillsboro, Missouri, and Lamar, Colorado, the direct path in the present AUTOVON connectivity) and to Rockdale, Georgia, (via Williamstown, Kentucky) for a total of five ports on the Toledo Junction CD. The Rockdale CD ties in the one local conferee for a total of two ports. The Cheyenne Mountain CD ties in its one local conferee and relays a call to Julian (via Socorro, NM) with three ports utilized at the Cheyenne Mountain CD. Julian ties in its two local conferees and relays on to North Bend (via Topaz Lake, NV) for a total of four ports utilized. Finally, North Bend ties in its local conferee for a total of two ports utilized.

A computer program was written to route all the conferees via this method and determine the accumulated requests for ports at each CD in the network. Note that the network topology, particularly CD placement and conference traffic matrix, is used to directly determine the arrival rate of conference calls as well as the probability distribution of the number of ports requested by a particular call. So the traffic characteristics at each CD of the network are different. The point should also be made here that we assumed that a conference was conducted even if not all the conferees could be connected, for whatever reason.

We now know the traffic loading on each CD in the network. The next step in this run is to determine the number of ports required to meet a desired loss probability. We have discussed in the introduction, and it should be clearer now, that a particular call may request more than one port. In fact, the minimum request is for two ports. A queueing model was developed to predict the behavior of this system and is described in Appendix B, but several points need to made here.

First, a call requesting two ports sees a different blocking probability than one requesting three ports, for instance; the reason is simply seen when one considers the situation where there are only two free ports and a call arrives. If a call requires two ports it gets in; if it requires three it does not. Secondly, because of this fact one is forced to consider an overall average loss probability for all calls using the particular CD. All ports are sized for a P10 grade of service. The total number of ports in the network and various other statistics are accumulated.

The final step in this run is to determine the effect this particular number of CD's has on CONUS AUTOVON. There are two main quantities that are used to reflect this impact. The first is the IST mileage used by the particular set of conference requirements. This measure is the number of IST miles which are utilized in processing the set of conferences, using the minimum spanning tree routing discussed earlier. We note this measure is not the additional IST channel miles required to support the conference traffic, but just the AUTOVON IST mileage that would be traversed. This mileage is considered in two functionally separate categories: USER/CD and CD/CD IST mileage. The first category represents (1) the IST mileage from the originator's home VON switch to his local CD, and (2) the IST mileage to each conferee's home VON switch from his local CD. For purposes of this study, this category (2) does not include any AUTOVON access line (PBX to switch) mileage since emphasis here is on the AUTOVON backbone. The second category is the CD to CD IST mileage, which represents the branches of the minimum tree spanning all CD's involved in connecting a specific conference.

The second impact to be determined is the additional AUTOVON trunking that would be required to maintain a desired grade of service within AUTOVON. This is accomplished by constructing a switch-to-switch traffic matrix, in erlangs, which accurately represents the conferencing requirements during a busy hour, on a call by call basis. It is also possible to add, or overlay, this conference traffic matrix onto the existing clear voice traffic matrix and to conduct a fixed-performance cost comparison of the AUTOVON network, both with and without the subject conferencing requirements. Using the DCEC Switched Network Design and Performance Model [7], one can determine this impact.

In the performace mode of this model, the identity of the 54 CONUS/CANADA AUTOVON switches and traffic flow between them is provided, along with the present design connectivity of the network. An average network point-to-point grade of service requirement of P10 is held fixed and the links are resized to meet this grade of service. The cost of the network before and after the inclusion of conference requirements is determined.

III. SYSTEMS ANALYSIS

In the first two sections of this technical note we have described the problems to be addressed and the techniques we have used to get answers to some basic questions. This section presents some numerical results of the study. Several tables and figures are used to display our results.

Table I shows the results of placing a variable number of CD's in the CONUS AUTOVON network, and Figures 5 and 6 show graphically the configurations in Table I corresponding to 2 and 10 CD's respectively. In Table I, each column represents a separate and independent run of the procedure described earlier. That is, there is no influence of one run upon another; the selection of a switch for one configuration, say four CD's, has no effect on its selection in another configuration. The selection, as mentioned earlier, is solely influenced by effort to minimize the total traffic weighted distances from 251 SVIP locations to the best selection of switches. Figures 5 and 6 show straight line segments having as one terminal point the SVIP location and as the other terminal point the associated CD location. Homing is done on the basis of closest CD. Not all 251 locations are visible in the resolution of these two figures.

As was suggested in section II, one measure of impact on CONUS AUTOVON is the Interswitch Trunk (IST) mileage. This measure is the AUTOVON trunk mileage traversed by a particular set of conferencing requirements. There are two components of this measure of special interest: first, the USER/CD mileage, the mileage of each call to the closest CD; and the second is the CD/CD, the CD to CD mileage for each call. In general, as the number of CD's increases, the user/CD mileage ratio for the set of 48 CD's decreases due to the shorter distances and greater proximity of users to their local CD's. Whereas, the CD/CD mileage for the same set of requirements will increase with more CD's in the system because as more CD's become available for use they will be utilized by the routing discipline. This phenomenon will occur only up to a certain point where additional CD's added to the system will not be as fully utilized by the conferences due to an upper limit on the number of conferees in a conference. Hence, a leveling off point is reached. The total IST mileage is thus the sum of two opposing monotonic functions of the number of CD's in the system. The relative change in one component will work against the other component and the sum will depend upon which relative change is larger.

Figure 7 shows the results of each component of IST miles as a function of the number of CD's present in the system for the three sets of 48 conferences. The three sets were generated using different random number seeds. As a result, both the number of conferees for a given conference and the distribution over the 251 locations of the conferees of a given conference were varied to examine the effects upon IST mileage used. The results of Figure 7 indicate that the effect of varying the simulation sampling for

TABLE I. SELECTION OF CD LOCATIONS FOR VARIABLE NUMBER OF CD EQUIPMENTS

AUTOVON SWITCH	Number of CD's in the Network									
	2	4	6	8	10	12	14	16	18	20
Apache Junction								X	X	X
Arlington		X	X	X	X	X	X	X	X	X
Brewton						X	X	X	X	X
Cedar Brook							X	X	X	X
Cheyenne Mountain				X	X	X	X	X	X	X
Delta										X
Dranesville	X									
Fairview						X			X	X
Hillsboro			X		X	X	X	X	X	X
Julian				X	X	X	X			
Littleton					X	X	X	X	X	X
Lodi				X	X	X	X	X	X	X
Mojave		X	X					X	X	X
Moseley								X	X	X
Mounds		X					X	X	X	X
North Bend			X	X	X	X	X	X	X	X
Polk City									X	X
Rockdale		X	X	X	X	X	X	X	X	X
Seguin				X	X	X	X	X	X	X
Socorro	X									
Stanfield										X
Sweetwater			X							
Terre Haute				X	X					
Toledo Junction						X	X	X	X	X
Wheatland							X	X	X	X

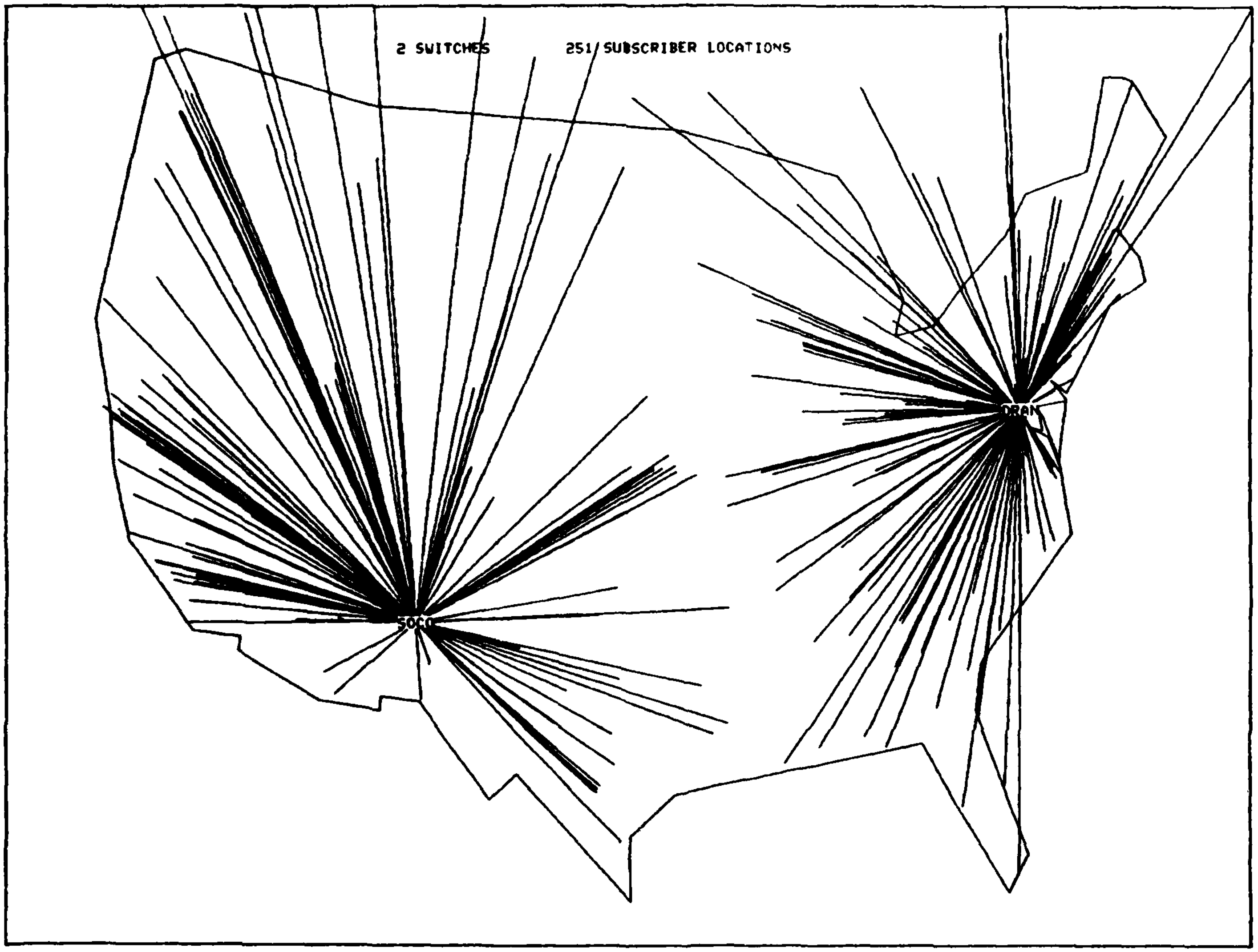


Figure 5. Selection of Two CONUS/Canada Switching Sites for CD Placement

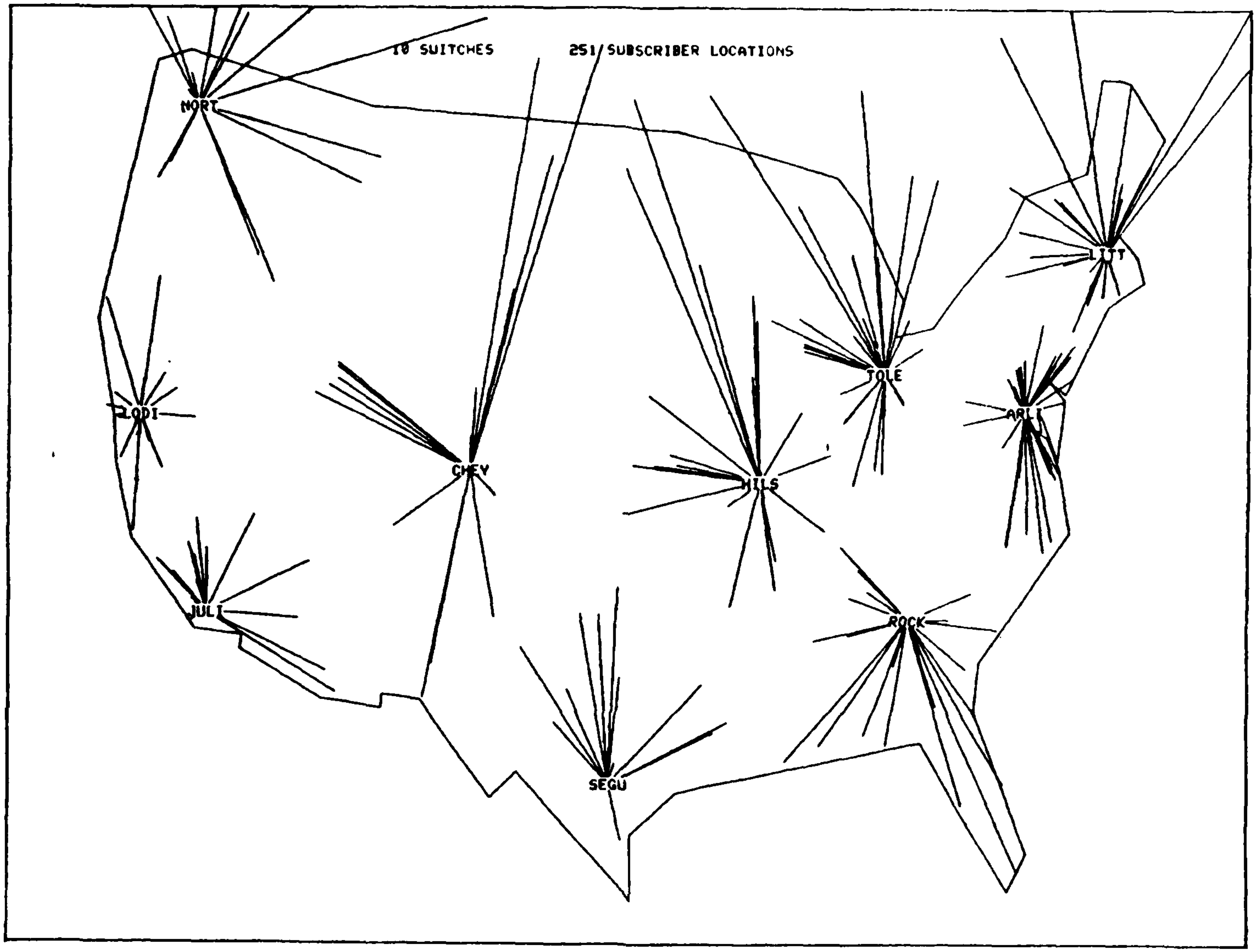


Figure 6. Selection of Ten CONUS/Canada Switching Sites for CD Placement

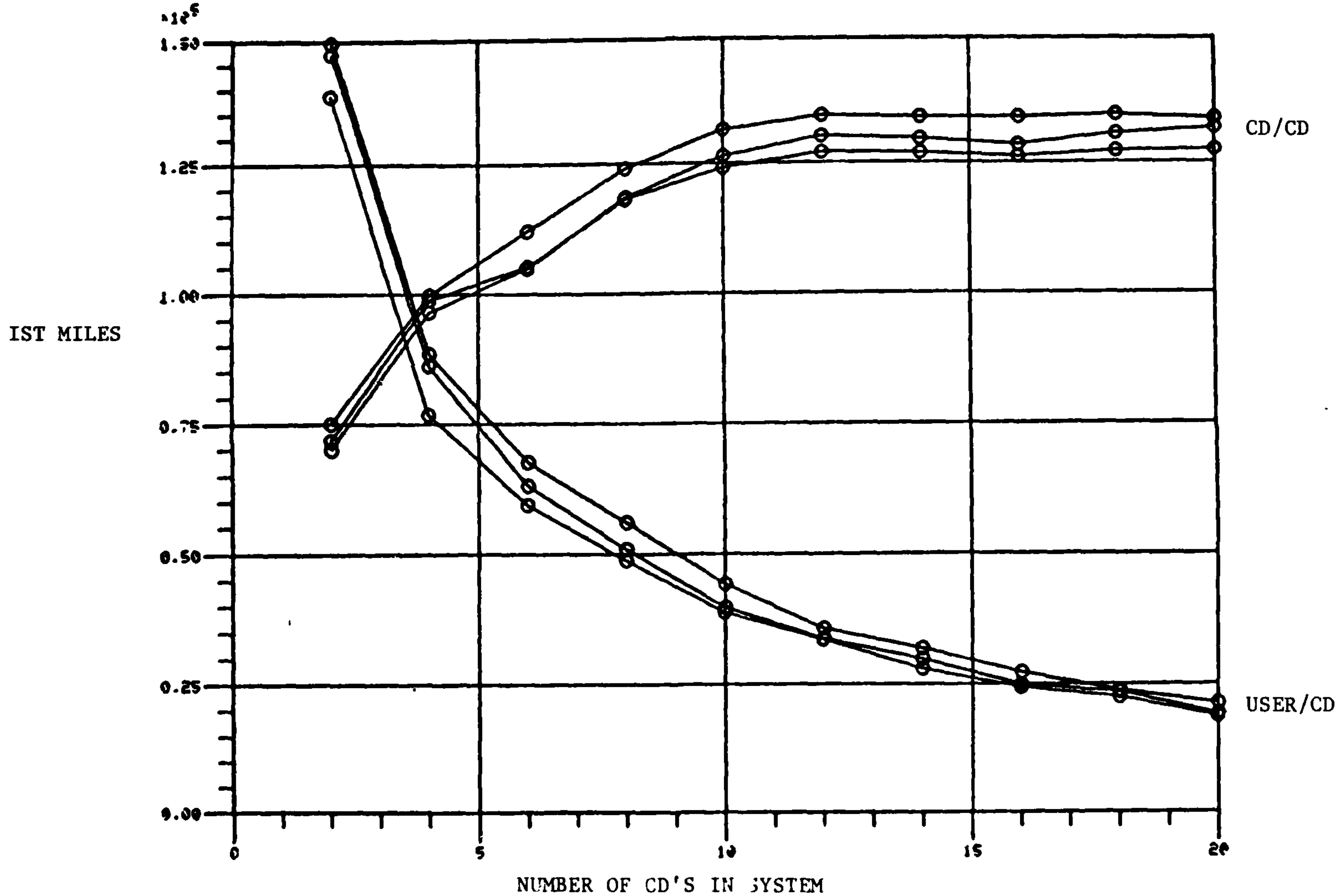


Figure 7. IST Mileage for Three Generations of a 48 Conference Requirements Set

number of conferees and location of conferees for a fixed set of originators is quite small. Since there was little difference in the three runs, in the remainder of the study the various figures of merit will be shown as averaged over the three separate sets of requirements. Also shown in Figure 7 is the leveling off of the CD/CD mileage at about 12 CD's, while the user/CD IST mileage is still decreasing at 20 CD's. Figure 8 shows the two types of IST mileage averaged and then summed as a function of number of CD's in the system. There is an initial drop in total mileage in going from two to four to six CD's, and from this point onward there is an overall trend of very slight decline in mileage. The initial sudden drop is due to the great effect of gaining access in the network to the local CD's of the users. This effect continues to dominate the increase in CD to CD mileage, but the combined effect after about six CD's is one of a diminishing effect on the mileage dropoff. Based solely upon this figure of merit, an optimally chosen set of six CD's would appear to have greatest impact upon use of IST mileage.

IST mileage, however, is not the only variable to be taken under consideration. In general the conferencing erlangs will be combined with clear and secure voice two party erlang traffic, and considerations of routing and economies of scale in trunk group sizing need to be examined. This is done in Table II for the 48 and 210 conferences per busy hour cases. For each case the additional cost of increasing the AUTOVON trunks to support the additional conference traffic and maintain a network grade-of-service of P10 is presented. The baseline case is when the number of CD's is zero.

TABLE II. COST OF AUTOVON TRUNKS TO SUPPORT THE ADDITIONAL CONFERENCING TRAFFIC

	48 Conferences		210 Conferences	
Number of CD's	CD Traffic (erlangs)	Cost (M$/mo.)	CD Traffic (erlangs)	Cost (M$/mo.)
0	0.0	6.310	0.0	6.310
2	60.22	6.353	262.44	6.500
4	60.89	6.346	268.83	6.469
6	60.78	6.343	274.16	6.470

Table II indicates a differential cost increase of 43, 36, and 33 K$/month for a network carrying 48 conferences with two, four and six CD's respectively. There is a slight cost savings (7 K$/mo) in going from two to four CD's and insignificant savings in going to six from four. From the network (and only the network) point of view, the selection of number of CD's is practically insensitive to cost considerations. The major cost considerations will more likely be found to lie within the CD system cost itself. Cost analysis for the CD equipments will be treated in subsequent reports. Increasing the number of conferences to 210 (a 4.4 fold increase), results in a differential cost increase of 190, 159, and 160 K$/month for the two, four, and six CD configurations. These bear an almost exact ratio to the 4.4 fold increase in traffic. In particular, the same conclusions apply to the number of CD's in the network. An initial configuration of two CD's will

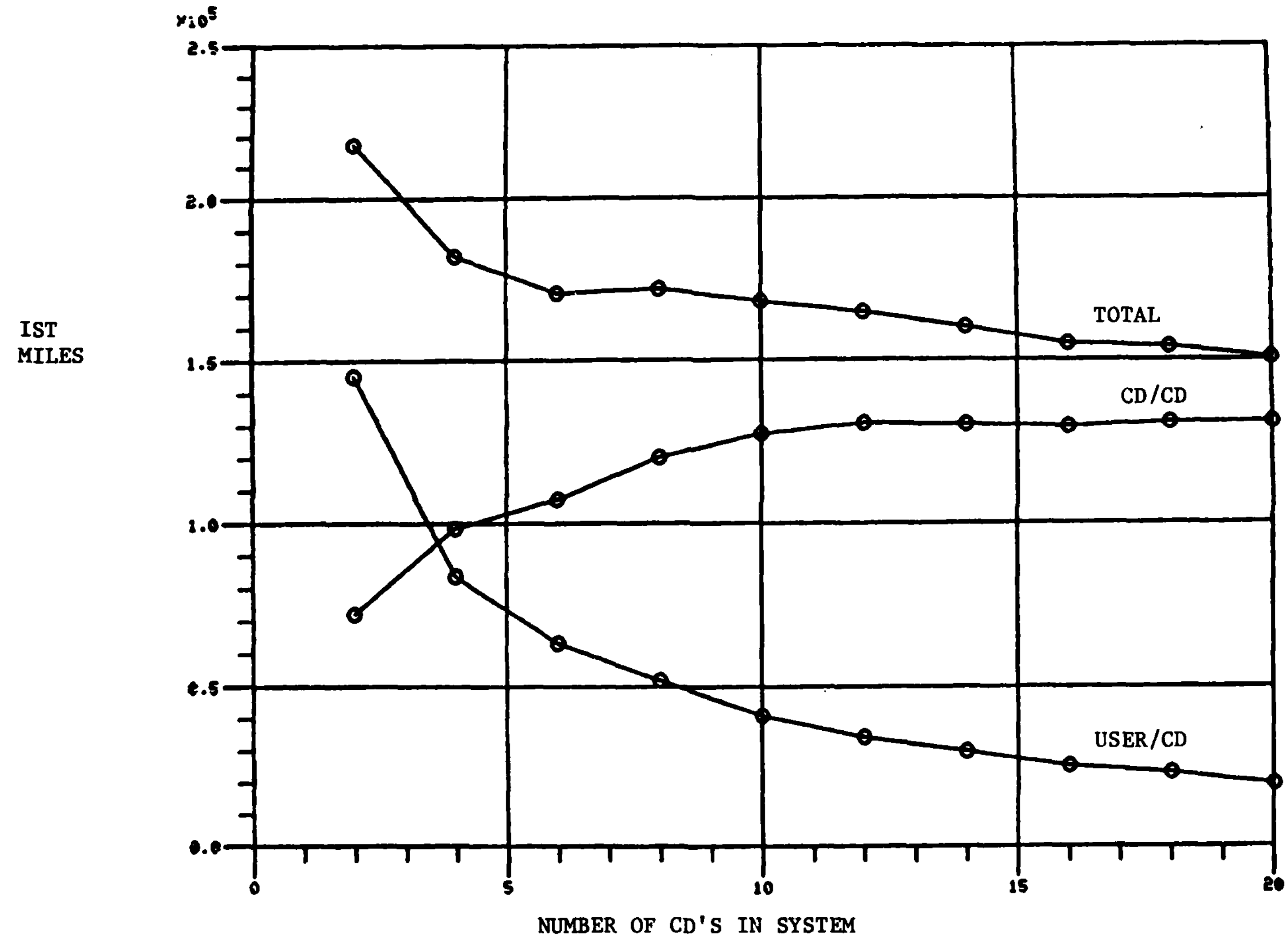


Figure 8. IST Mileage Vs. Number of CD's (48 Conferences)

be slightly more expensive to AUTOVON than four, and in each case this cost will vary in a direct linear manner with numbers of conferences (i.e., volume of requirements).

A CD collocated with an AUTOVON switch will see both incoming and outgoing transactions regarding requests for its ports. First, on the incoming side, a single port will be seized from either a call from a conference originator (user) on the local CD or from another CD that is relaying the conference because it has conferees local to it. Then, on the outgoing side, groups of ports will be seized for calls to (1) each conferee involved in the conference, and (2) CD's involved in further tandeming of the conference. For a given conference, a specific CD may see no such transactions at all, even though its associated switch may be involved in the conference. Thus, a CD for one specific conference will see either no requests at all for ports or a request for two, three, or more ports, depending on its proximity to the users in the conference and the presence of other CD's in the network.

Figure 9 shows the number of conferences (a single request by a conference) being offered to a typical CD in the network versus the number of CD's in the network. Note that a request could result in any number of requested ports, but here only individual requests are counted. If the number of CD's in the network were one, the number of transactions would be 48, the number of conferences in the requirements data base. This shows how the conference requests on a CD drops as there are more CD's in the network. An average CD will, up to a point, see fewer and fewer conferences requesting service from it. Figure 10 shows the total number of ports being requested by an average CD. Both Figures 9 and 10 show that as more and more CD's are added to the system, with the requirements held fixed there will be less and less activity present at an average CD up to a point of diminishing returns due to geographical separation from the requirements flows. This emphasizes the need for accurate knowledge of conference flow patterns.

Figure 11 shows the average number of CD's that will be utilized by a typical conference. When only two CD's are present in the system, nearly all (1.9 on the average) will be utilized by an average conference. This ratio falls off rapidly, however, when more and more CD's are added to the network. For example, with 20 CD's in the network, only about 5.4 CD's (on the average) are utilized by a given conference. This is directly related to and controlled by the distribution of the number of conferees in a conference.

Figure 12 shows the number of ports requested by an average conference as more and more CD's become available in the network. Up to a certain point, more and more ports are being requested. This is due to the fact that more and more CD's are being involved in a conference because of their geographic availability. With more CD's available, the number of overall ports requested increases. This effect diminishes due to a saturation effect; i.e., additional CD's will tend not to be involved in a given conference. The conclusion is that from the conference point of view, the inclusion of more and more CD's in a network will have diminishing effect due to the upper limit on numbers of conferees in a conference. These figures have demonstrated that the optimal number of CD's in the CONUS AUTOVON is probably small and on the order of two to six CD's, depending on the location of the users.

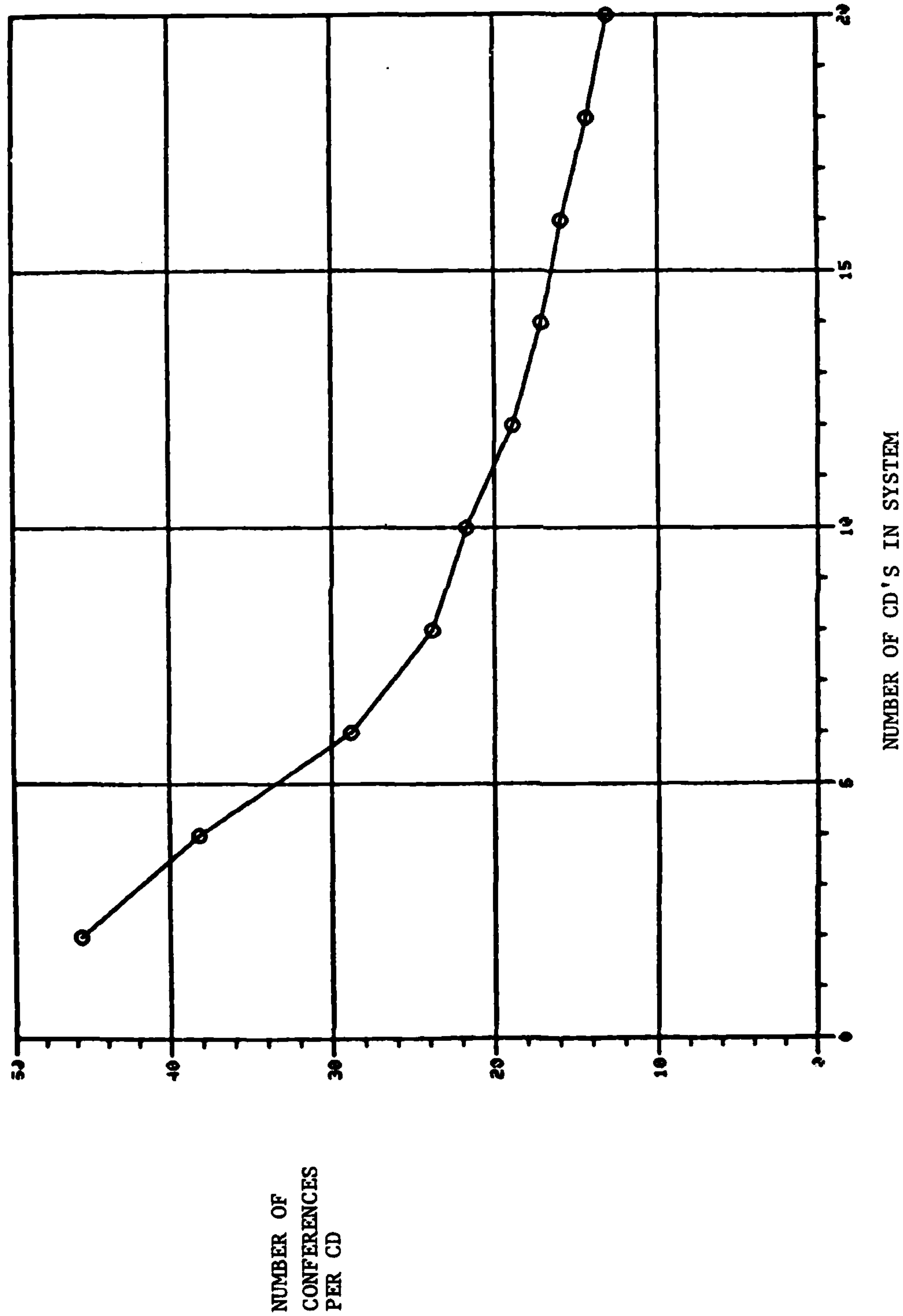


Figure 9. Conference Activity for an Average CD

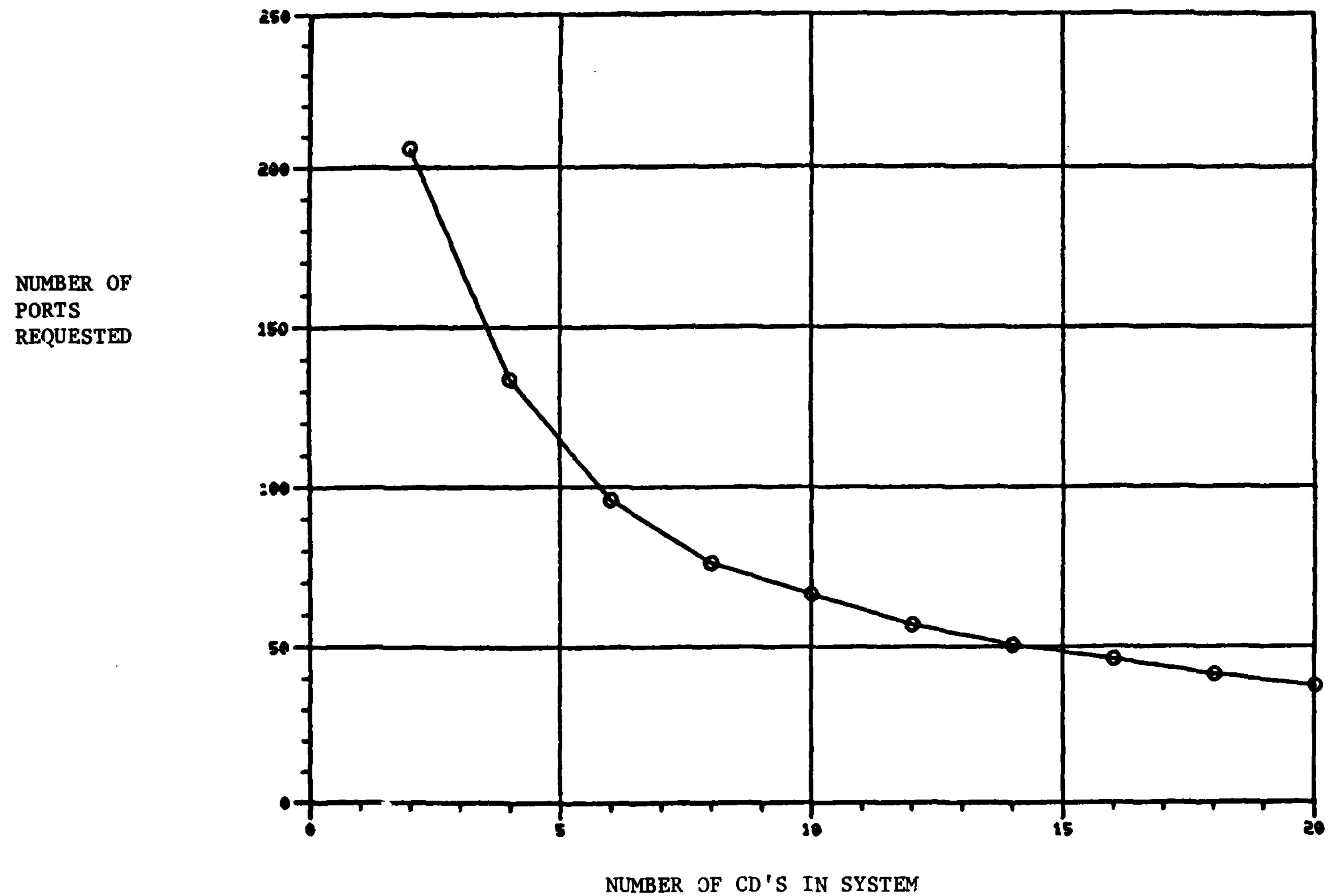


Figure 10. Port Request Activity for an Average CD

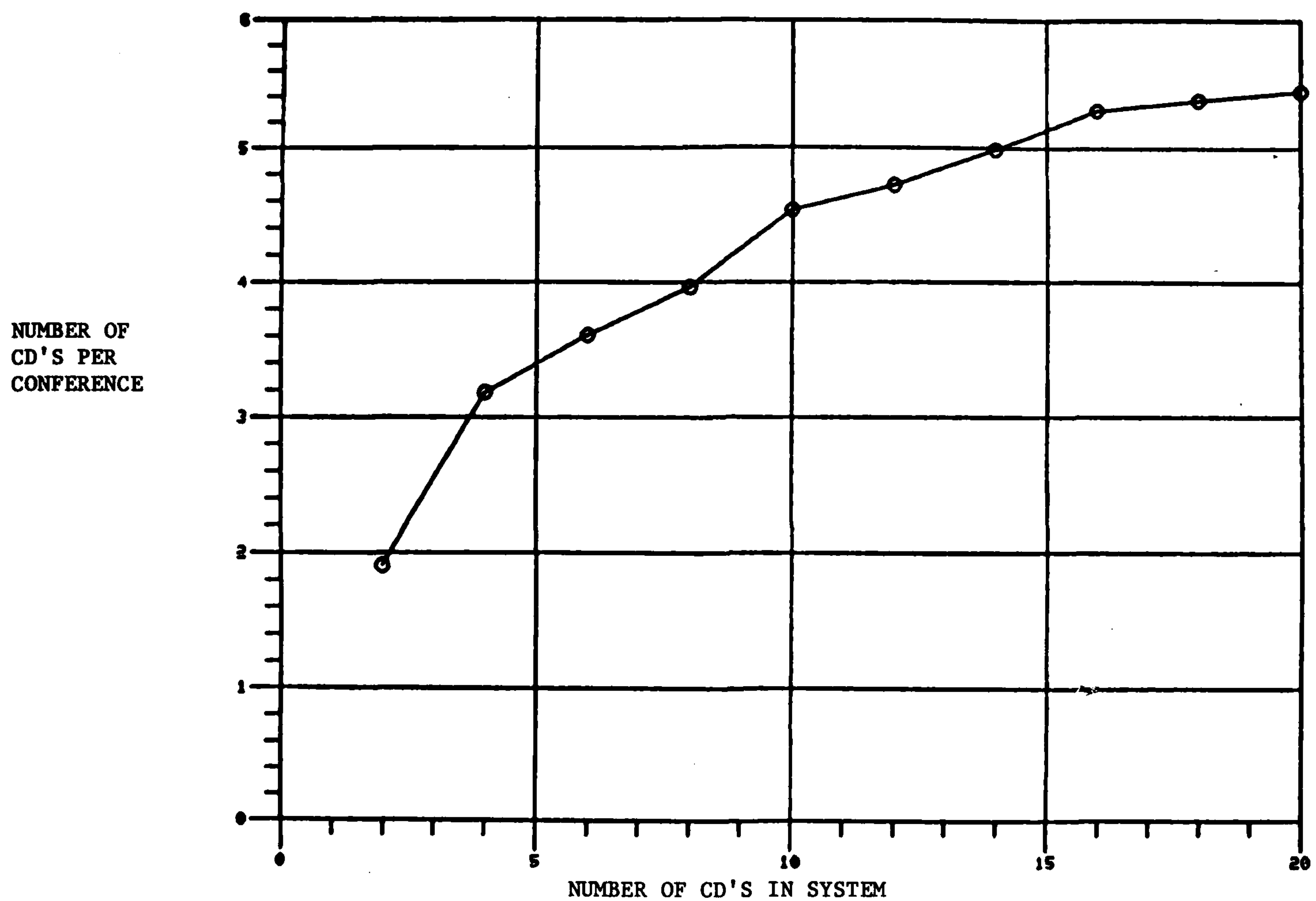


Figure 11. Number of CD's Utilized by a Typical Conference

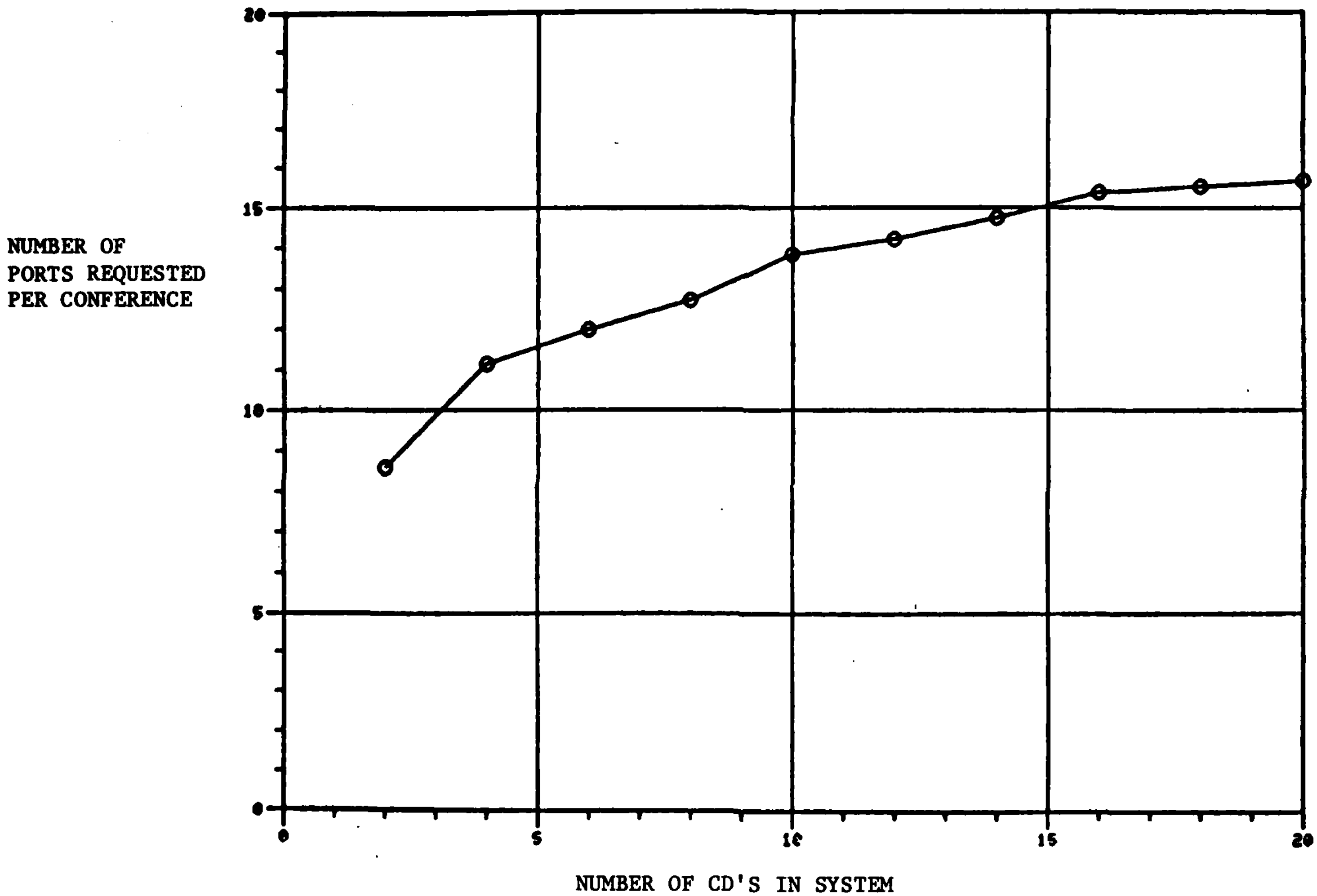


Figure 12. Ports Requested by an Average Conference

The previous figures have shown the process of requests for ports flowing through an AUTOVON network configured for a variable number of CD's. Any number of ports on a CD less than the total requested over a busy hour will lead to blocking on the CD, or denial of service. Appendix B discusses a mathematical queueing model of this process whereby the number of ports associated with a desired level of blocking can be evaluated (by a computer algorithm) from a knowledge of the distribution of numbers of ports requested for a given CD. This sizing in effect allows for more efficient use of the resources (ports) on the CD. No limitation is assumed in the study on the number of conference bridges available to the CD.

Table III shows the distribution of the ports on each CD in the system as determined by the queueing model described in Appendix B. These quantities were averaged over the three separate traffic simulations for the case of 48 conferences in the system and grade of service on the CD (blocking) of P.10.

It has already been seen that as more CD's are available to the network, they are utilized less and less frequently. This will also cause the number of ports, sized for an assumed level of blocking at the CD of P10, to decrease at a CD. The question is whether the total number of ports in the network will also decrease with increasing quantities of CD's. Figure 13 shows the total number of ports, sized for P10 CD blocking, for each configuration. This figure indicates that although the ports per CD are decreasing, the number of CD's is rising faster, resulting in an increase in the number of ports in the system. This is more than likely due to the total lack of economy of scale at the lighter loading levels produced with many CD's in the system. Adding CD's will, in general, result in more ports overall in the system.

TABLE III. DISTRIBUTION OF CD PORTS FOR VARIABLE NUMBER OF CD'S IN THE NETWORK (P.10 BLOCKING)

AUTOVON SWITCH	Number of CD's in the Network									
	2	4	6	8	10	12	14	16	18	20
Apache Junction								10	10	10
Arlington		43	39	38	36	36	34	24	27	28
Brewton						14	14	15	14	14
Cedar Brook							13	13	13	14
Cheyenne Mountain				16	15	15	15	13	13	12
Delta										7
Dranesville	57									
Fairview						12			9	9
Hillsboro			28		24	21	22	21	18	18
Julian				18	19	18	19			
Littleton					13	13	12	12	12	12
Lodi				14	14	14	14	11	11	11
Mojave		26	24					18	18	18
Moseley								19	19	16
Mounds		34					15	15	15	16
North Bend			8	8	8	8	8	8	8	8
Polk City									3	3
Rockdale		28	26	27	24	18	18	18	18	15
Seguin				20	19	19	14	14	11	14
Socorro	38									
Stanfield										14
Sweetwater			25							
Terre Haute				27	18					
Toledo Junction						19	19	18	18	18
Wheatland							6	5	5	6

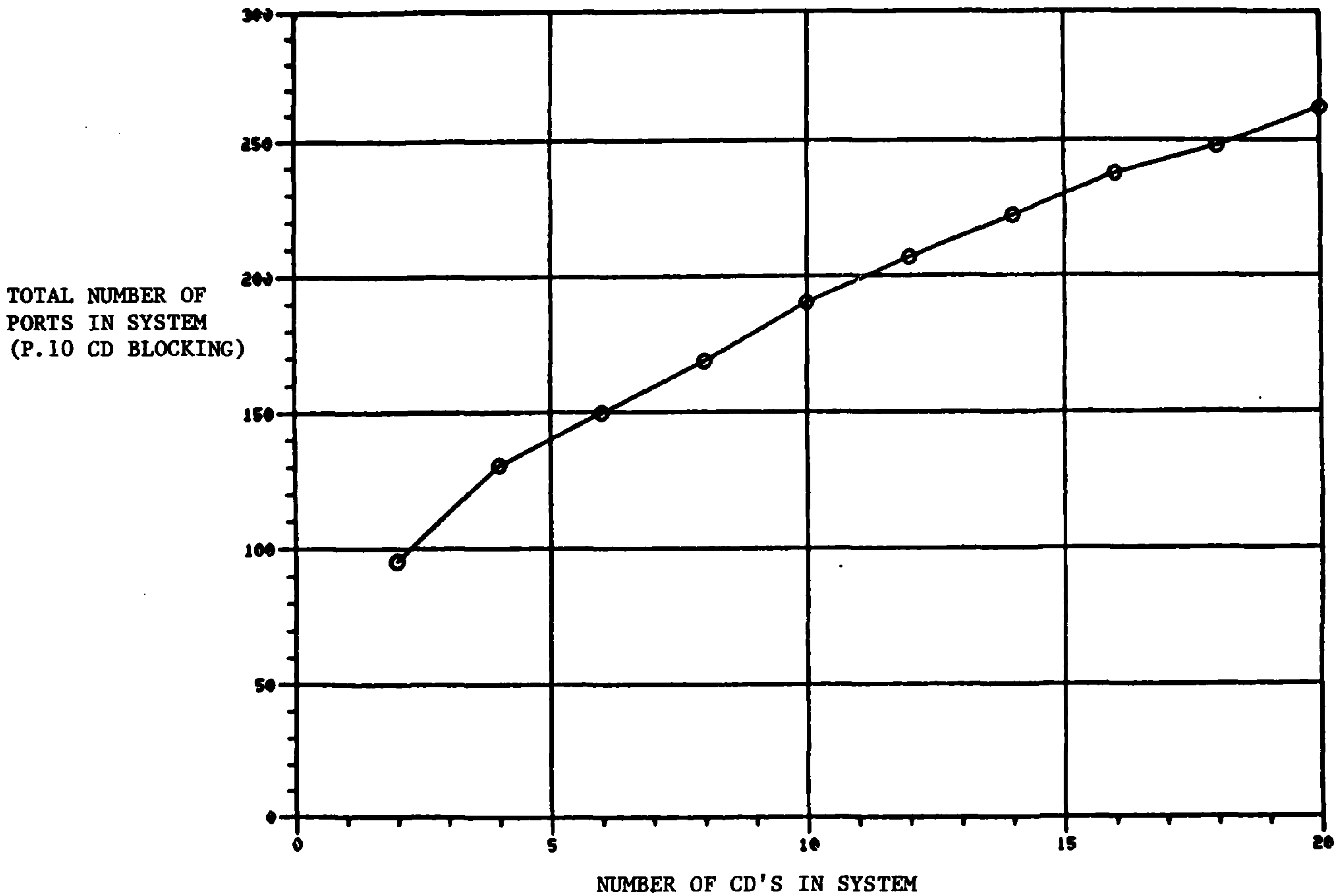


Figure 13. Total Number of Ports in System (P.10 Blocking on CD)

IV. SIGNIFICANT FINDINGS AND CONCLUSIONS

In this Technical Note we have studied the impact on CONUS AUTOVON of the addition of estimated SVIP conferencing requirements. This effort required two significant pieces of developmental analysis. The first was the generation of a representative traffic requirements matrix for SVIP conference calls; and the second was a development of a mathematical model to be used in sizing the ports on the CD. These two pieces of work are fully documented in Appendices A and B.

The study attempted to answer the following questions:

a. How many CD's should there be in CONUS AUTOVON?

b. Where should they be located?

c. What is the port sizing required to meet the conference traffic requirements?

d. What is the impact of accommodating the conferencing traffic requirements on AUTOVON?

A detailed analysis of these questions is given in section III; but some general statements can also be made. The number and locations of the CD's are highly dependent on the location and traffic requirements of the users. When one only considers the traffic requirements generated in this study, the optimal number of CD's is somewhere between 2 and 4. This result only considers the network implications of the CD's and their traffic; it did not consider the cost of the CD's themselves. When one considers the economies of scale in the port sizing aspects of the problem, and the cost of the CD's, one is forced to conclude that the optimal number of CD's is probably very small, probably two, considering cost impact. The survivability/reliability aspects of the problem have not been addressed in this report but could obviously impact the above conclusions.

REFERENCES

[1] Letter dated 22 Feb 80 from DCA Headquarters Code 440, Titled, "Tasking to Perform Network Configuration Analysis and Life Cycle Cost Analysis for SVIP."

[2] Letter dated 9 May 80 from DCA Headquarters Code 440, Titled, "Clarification of 22 Feb 80 Letter."

[3] R. B. Cooper, Introduction to Queueing Theory, MacMillan, New York, 1972.

[4] S. Lin, "Computer Solutions of Traveling Salesman Problems," BSTJ, December 1965.

[5] S. Lin and B. W. Kernighaw, "An Effective Hueristic Algorithm for the Traveling Salesman Problem," Operation Research 21 No. 2, March-April 1973.

[6] F. S. Hillier and G. J. Lieberman, Introduction to Operations Research, Holden-Day, Inc., San Francisco, California, 1967.

[7] M. J. Fischer, D. A. Garbin, T. C. Harris, and J. E. Knepley, "Queueing Network Problems in Large Scale Communications Systems," OMEGA, 6, No. 3, 1978.

APPENDIX A
GENERATION OF SVIP CONUS CONFERENCING TRAFFIC REQUIREMENTS

1. Introduction. In order to perform network analyses to determine the numbers, placements and sizing of SVIP Conference Directors within the CONUS AUTOVON network, it is necessary to have a set of busy hour conferencing traffic requirements. These traffic requirements should be specific enough to indicate the quantities of busy hour conferences, the locations where they originate, the numbers and locations of their conferees in a conference by conference basis and the holding time of a conference. In the absence of a set of detailed quantified requirements as described above, it has been necessary to generate, using Monte Carlo simulation techniques, an interim set of conferencing requirements. This appendix discusses the procedure, rationale and inherent assumptions made in arriving at an interim set of requirements. In the implementation of this procedure, certain parameters and probability distributions were subjectively chosen on the basis of a first cut estimation.

The basic data base from which these procedures derive is a set of 251 CONUS/Canada locations with busy hour originating SVIP (two party) offered traffic in erlangs. This data base was provided to R730 in the first quarter of CY 78 by R720 and is the result of an earlier analysis which used as input the CONUS/Canada portion of the CY 78 Secure Voice 10,000 subscriber list, and which derived originating offered traffic by location. This data base was used to provide CONUS secure voice traffic for use in the 1982-92 Ten Year Plan. It is presumed that prior analysis considered the numbers of subscribers (main stations, or instruments) at a location, an assumed amount of originating traffic per instrument (about 1.2 calls per busy hour), a turn-around ratio (about 70%) at the local concentrator and a sizing of access lines according to a desired blocking probability (about 10%). The resulting offered erlang traffic is thus a derivative of the locations of main stations in the CY 78 Secure Voice data base. A listing of the 251 locations is given in Figure A-1. Each line is a SVIP location and shows the location number, the 8 character DCA geographical location name, a 2 character state/country code, decimal latitude and longitude, V/H coordinates, and SVIP (two party) offered erlangs. The total traffic over the 251 locations was 545 busy hour erlangs.

The method for using this data to simulate SVIP conferencing requirements consists of three steps:

1. Determine the number of busy hour conferences originating from each location.

2. For each conference in step 1, determine the number of conferees participating in the conference.

3. Determine the location of the conferee in step 2.

LOC#	LOC NAME	LAT	LONG	V	H	SVIP TRAF IN ERLANGS
001	ABERDEEN24	39.492	70.136	5429	1533	3.465
002	ALAMEDA 06	37.783	122.267	8491	8695	0.285
003	ALBANY 36	42.667	73.800	4641	1638	0.543
004	ALEXANDR51	38.817	77.050	5639	1575	1.668
005	ALGTNHLS51	38.867	77.100	5635	1588	1.222
006	ALTUS 40	34.667	99.267	8220	4602	3.673
007	ANDREWS 24	38.811	76.867	5622	1549	1.550
008	ARCATA 06	40.983	124.100	7815	9067	0.069
009	ARGENTIACA	47.310	53.991	1772	199	0.170
010	ARLINGTN51	38.917	77.083	5624	1592	6.837
011	ARNOLD 47	35.333	86.083	7100	2522	0.343
012	ATLNTCCY34	39.367	74.450	5284	1287	0.554
013	AUGUSTA 23	44.333	69.750	3955	1370	0.594
014	BANGOR 53	47.800	122.600	6297	8938	0.063
015	BARKSDAL22	32.500	93.567	8267	3480	2.163
016	BARSTOW 06	34.900	117.000	8994	7686	0.240
017	BATTLECR26	42.317	85.183	5713	3124	0.897
018	BEALE 06	39.136	121.436	8181	8586	0.968
019	BEAUSEJRCA	50.067	96.550	4963	5411	0.189
020	BEAVRLDGCA	55.217	119.433	4630	8623	0.109
021	BERGSTRM48	30.195	97.653	9014	3976	5.059
022	BIRMNGHM01	33.517	86.817	7519	2447	0.343
023	BLDYHGHSCA	53.617	122.950	5018	9052	0.109
024	BLYTHVLL05	35.964	89.946	7304	3188	1.189
025	BOTHELL 53	47.762	122.203	6301	8879	0.284
026	BROOKS 48	29.346	98.432	9239	4046	0.278
027	BRUNSWCK23	43.900	69.967	4050	1333	1.806
028	BUCKHNNN54	39.000	80.233	5905	2047	0.277
029	C DYER CA	66.617	61.300	-181	4244	1.383
030	CANNON 35	34.384	103.317	8518	5270	1.071
031	CARLSLBK42	40.200	77.167	5401	1764	3.816
032	CARSNCY32	39.192	119.735	8135	8301	0.507
033	CARSWELL40	32.769	97.437	8483	4143	1.155
034	CASTLE 06	37.331	120.568	8547	8392	0.709
035	CHANUTE 17	40.292	88.144	6330	2338	0.468
036	CHARLSTN45	32.800	79.950	7020	1283	4.802
037	CHATHAM CA	42.400	82.183	5451	2716	0.080
038	CHERRYPT37	34.900	76.883	6329	1067	0.472
039	CHIBOUGMCA	49.950	74.350	3431	2692	0.157
040	CHINA L 06	35.637	117.689	8845	7837	0.851
041	CHRLTSVL51	38.033	78.483	5918	1682	2.215
042	CHYNNMTN08	38.317	104.717	7679	5795	4.247
043	CLEVELND39	41.500	81.683	5574	2543	3.260
044	COLUMBUS28	33.500	88.450	7660	2709	0.686
045	COLUMBUS39	39.967	83.000	5972	2555	0.796
046	COMOX CA	49.667	124.917	5905	9291	0.109
047	CONCORD 06	37.983	122.050	3444	8662	0.069
048	CORONADO06	32.700	117.200	9473	7638	6.572
049	CP DAVID24	38.817	76.867	5621	1550	0.203
050	CP DRUM 36	44.050	75.733	4582	2067	0.820
051	CPDOUGLS55	43.917	90.267	5787	4003	0.336
052	CPLEJEUN37	34.667	77.350	6419	1108	1.104
053	CPMURRAY53	47.117	122.567	6447	8923	0.532
054	CPPICKTT51	37.050	77.933	6044	1483	0.191
055	CPPNDLTN06	33.317	117.367	9346	7692	6.961
056	CPROBRTS06	35.832	120.744	8865	8379	0.175
057	CRANE 18	38.900	86.900	6498	3011	0.469
058	CRPSCHRS48	27.800	97.400	9475	3739	0.261
059	DANA CA	52.283	105.767	4923	6766	0.116
060	DAYTON 39	39.750	84.200	6114	2704	8.001
061	DOVER 10	39.167	75.533	5428	1409	3.358
062	DUGWAY 49	40.200	112.933	7731	7211	1.182
063	DVSMNTHN04	32.166	110.882	9354	6469	3.296
064	DYESS 48	32.417	99.850	8712	4531	3.431
065	EARLE 34	40.017	74.600	5185	1392	0.313
066	EDMONTONCA	53.550	113.467	4885	7821	2.023
067	EDWARDS 06	34.905	117.883	9018	7847	2.979
068	EGLIN 12	30.433	86.500	8071	2087	9.612
069	ELLSWRTH46	44.146	103.104	6500	5880	1.977
070	ENGLAND 22	31.333	92.550	8412	3117	0.570
071	FAIRCHLD53	47.625	117.650	6258	8210	2.392
072	FEWARREN56	41.150	104.800	7200	5956	1.907
073	FLCNBRDGCA	46.617	80.833	4565	3034	0.075
074	FORBES 20	38.952	95.662	7129	4359	0.117
075	FRANKFRT21	38.217	84.833	6456	2631	0.090
076	FRESNO 06	36.783	119.750	8659	8233	0.175
077	FT BLISS48	31.800	106.417	9218	5646	4.365
078	FT BRAGG37	35.133	78.983	6496	1408	8.981
079	FT DIX 34	40.017	74.550	5180	1385	1.938

Figure A-1. Listing of SVIP Locations

080	FT HOOD 48	31.133	97.767	8833	4070	7.001
081	FT KNOX 21	37.900	85.983	6613	2770	1.504
082	FT LEWIS53	47.083	122.600	6455	8928	2.465
083	FT MCCOY55	43.150	90.133	5926	3910	0.336
084	FT MEADE24	39.100	76.833	5567	1580	6.140
085	FT ORD 06	36.650	121.767	8730	8580	5.351
086	FT POLK 22	31.046	93.192	8518	3271	0.911
087	FT RILEY20	39.067	96.783	7178	4544	3.032
088	FT SILL 40	34.650	98.402	8168	4455	7.063
089	FTBELVCR51	38.683	77.133	5671	1570	0.584
090	FTBENNNG13	32.383	84.883	7564	2018	2.832
091	FTBNHRSN18	39.850	86.017	6246	2980	0.936
092	FTCARSON08	38.733	104.800	7700	5803	4.144
093	FTCMPBLL21	36.667	87.483	6972	2872	3.542
094	FTDETRCK24	39.433	77.433	5565	1705	0.610
095	FTGILLEM13	33.583	84.350	7289	2060	1.816
096	FTGORDON13	33.417	82.133	7116	1663	1.816
097	FTHUACHC04	31.580	110.340	9454	6339	5.318
098	FTJACKSN45	34.050	30.933	6884	1578	7.095
099	FTLNRDWD29	37.733	92.117	7124	3696	1.418
100	FTLVNWRT20	39.350	94.917	7000	4276	0.700
101	FTMCCLLN01	34.717	85.783	7201	2406	0.687
102	FTMCPHSN13	33.700	84.417	7273	2083	10.320
103	FTMCNMTH34	40.300	74.050	5080	1356	3.888
104	FTMONROE51	37.033	76.300	5886	1247	7.451
105	FTRITCHI24	39.733	77.417	5509	1740	10.511
106	FTSHERDN17	42.223	87.816	5935	3497	2.302
107	FTSMHSTN48	29.450	98.450	9218	4058	0.596
108	FTSTEWRT13	31.850	81.600	7357	1435	0.380
109	GANDER CA	48.950	54.567	1611	547	0.080
110	GEORGE 06	34.583	117.533	9078	7769	2.687
111	GLENVIEW17	42.083	87.817	5962	3472	0.329
112	GOODFLLW48	31.430	100.401	8947	4554	0.546
113	GR FORKS38	47.917	97.050	5421	5302	3.777
114	GRAYLING26	44.680	84.744	5234	3321	0.197
115	GRIFFISS36	43.233	75.407	4696	1918	5.121
116	GRISSOM 18	40.650	86.150	6105	3033	2.184
117	GROTON 09	41.317	72.200	4717	1250	2.578
118	GT L 17	42.302	87.818	5920	3495	2.016
119	GTWLNGTN10	39.678	75.607	5344	1485	0.277
120	GYPSMVLLCA	51.750	98.583	4727	5807	0.189
121	HARTFORD09	41.750	72.700	4692	1373	0.543
122	HAWTHORN32	38.527	118.625	8254	8094	1.015
123	HELENA 30	46.600	112.017	6336	7347	0.089
124	HILL 49	41.117	111.967	7503	7055	4.387
125	HOLBERG CA	50.517	128.017	5725	9727	0.109
126	HOLLOMAN35	32.850	106.100	8984	5655	1.575
127	HOMESTED12	25.483	800.467	8434	542	4.304
128	INDINPLS18	39.750	86.167	6277	2991	0.936
129	INDNTWGP42	40.367	76.633	5321	1712	0.080
130	JFFRSNCY29	38.592	92.156	6958	3781	0.177
131	JOHNSVLL42	40.199	75.066	5198	1479	0.519
132	KAMLOOPSCA	50.667	120.333	5639	8661	0.109
133	KANSASCY29	39.033	94.583	7041	4197	0.132
134	KELLY 48	29.367	98.567	9243	4072	18.564
135	KGSLYFLD41	42.163	121.736	7524	8705	0.507
136	KIRTLAND35	35.733	108.350	8284	6274	2.159
137	KISAWYER26	46.354	37.394	5116	3655	2.056
138	LA JUNTA08	37.983	103.511	7789	5544	0.220
139	LACKLAND48	29.400	98.600	9233	4081	0.131
140	LAKEHRST34	40.033	74.354	5157	1361	0.867
141	LANGLEY 51	38.950	77.167	5626	1609	2.822
142	LANSING 26	42.750	84.583	5533	3087	0.395
143	LAWRENCE20	38.967	95.250	7098	4295	0.233
144	LEWES 10	38.783	75.117	5454	1303	0.277
145	LEXINGTN21	38.083	84.500	6453	2567	0.226
146	LGHANSCM25	42.490	71.277	4422	1295	8.013
147	LITTLRCK05	34.733	92.250	7721	3444	5.961
148	LONGBECH06	33.750	118.233	9277	7864	4.866
149	LORING 23	46.950	67.896	3334	1540	0.966
150	LOSANGLS06	33.940	118.400	9241	7901	2.128
151	LOWRY 08	39.717	104.900	7502	5802	1.414
152	LOWTHER CA	49.550	83.000	4196	3649	0.075
153	LTTRKNNY42	39.933	77.650	5486	1803	1.111
154	LUKE 04	33.523	112.383	9130	6807	0.549
155	MACDILL 12	27.850	82.517	8197	1146	2.425
156	MADISON 55	43.083	89.367	5884	3794	0.336
157	MALMSTRM30	47.500	111.167	6119	7263	1.430
158	MARCH 06	33.893	117.250	9220	7692	7.674
159	MARE I 06	38.100	122.267	8422	8702	0.065
160	MAYNARD 25	42.500	71.550	4447	1330	0.581
161	MAYPORT 12	30.400	81.417	7610	1244	0.816

Figure A-1. Listing of SVIP Locations (Continued)

162	MCCHORD 53	47.148	122.479	6439	8911	5.717
163	MCCLELLN06	38.667	121.399	8282	8568	3.304
164	MCCONNLL20	37.617	97.267	7499	4503	3.142
165	MCGUIRE 34	40.033	74.583	5180	1392	6.473
166	MCNC3BRG42	40.217	77.017	5384	1745	0.203
167	MEMPHIS 47	35.117	90.083	7479	3129	0.149
168	MILWAUKE55	43.033	87.917	5787	3584	0.329
169	MINOT 38	48.267	101.317	5567	5914	1.889
170	MOODY 13	30.967	83.200	7676	1595	0.340
171	MORGNTWN54	39.650	79.917	5757	2079	1.111
172	MTN HOME16	43.050	115.867	7203	7795	0.311
173	N LONDON09	41.350	72.117	4703	1244	0.991
174	NASHVILL47	36.150	86.300	7014	2713	0.343
175	NATICK 25	42.283	71.350	4464	1275	0.211
176	NELLIS 32	36.233	115.033	8648	7395	3.298
177	NEWPORT 44	41.217	71.300	4642	1121	0.126
178	NO BAY CA	46.317	79.467	4506	2825	1.134
179	NORFOLK 51	36.667	76.233	5945	1191	11.641
180	NORLEANS22	29.967	90.117	8483	2647	4.441
181	NORTON 06	34.092	117.236	9175	7698	4.520
182	NWCMBLND42	40.217	76.900	5373	1730	0.967
183	NYORK CY36	40.717	74.017	5004	1406	6.091
184	OAKLAND 06	37.783	122.216	8490	8686	0.069
185	OCEANA 51	36.822	76.032	5896	1182	1.673
186	OFFUTT 31	41.133	95.917	6710	4580	5.090
187	OGDEN 49	41.233	111.967	7478	7100	0.394
188	OKLAHMCY40	35.467	97.533	7949	4375	2.939
189	ORANGE 09	41.283	73.033	4806	1353	0.271
190	OTTAWA CA	45.450	75.700	4332	2246	0.383
191	PASCAGOL28	30.350	88.533	8275	2415	0.142
192	PATRICK 12	28.238	80.603	7934	871	1.744
193	PEASE 33	43.033	70.817	4274	1321	2.436
194	PENTAGON51	38.867	77.050	5630	1581	15.906
195	PHILDLPH42	39.950	75.163	5252	1460	5.296
196	PICATNNY34	40.017	74.600	5135	1392	0.277
197	PLTSBRGH36	44.650	73.467	4265	1064	1.414
198	POPE 37	35.171	79.015	6492	1417	0.325
199	PORTHJNM06	34.150	119.217	9217	8055	0.426
200	PORTLAND41	45.500	122.617	6803	8906	0.566
201	PORTSMTH51	36.867	76.400	5925	1240	7.456
202	PSDSNFNC06	37.800	122.451	8491	8727	5.551
203	PT MUGU 06	34.117	119.117	9222	8036	1.198
204	PT REYES06	38.000	122.967	8455	8820	0.069
205	QUANTICO51	38.500	77.300	5720	1571	0.871
206	RANDOLPH48	29.529	98.279	9190	4033	1.558
207	RCPDSGBR29	38.851	94.553	7075	4177	3.265
208	REDSTONE01	34.617	86.667	7295	2535	8.928
209	REESE 48	33.600	102.033	8605	4995	0.247
210	RESTON 51	38.917	77.350	5650	1630	4.239
211	RICHMOND51	37.567	77.433	5901	1475	0.574
212	ROBINS 13	32.640	83.592	7397	1939	0.597
213	ROCK I 17	41.536	90.569	6270	3317	5.261
214	SALEM 41	44.933	123.017	6933	8959	0.532
215	SALTL CY49	40.767	111.883	7574	7066	0.394
216	SANANTON48	29.417	98.500	9223	4064	1.161
217	SANTARCS06	38.433	122.717	8356	8787	0.069
218	SCOTT 17	38.550	89.850	6756	3422	7.280
219	SENECA 36	42.900	76.967	4890	2064	0.353
220	SHARPE 06	37.833	121.267	8462	8524	0.138
221	SIERRA 06	39.500	120.500	8084	8438	2.537
222	SIOUXLKTCA	50.083	92.000	4701	4825	0.376
223	ST LOUIS29	38.650	90.417	6819	3519	4.418
224	ST PAUL 27	44.950	93.083	5774	4427	1.344
225	STOCKTON06	37.950	121.283	8437	8530	0.069
226	SYMJHNSN37	35.340	77.960	6357	1280	0.497
227	TOBYHANN42	41.183	75.417	5059	1553	0.229
228	TOOELE 49	40.533	112.317	7639	7125	1.182
229	TOPEKA 20	39.033	95.683	7114	4769	0.117
230	TRACY 06	37.750	121.433	8483	8551	0.069
231	TRAVIS 06	38.262	121.927	8380	8645	2.955
232	TRENTON 34	40.217	74.750	5164	1438	0.831
233	TYNDALL 12	30.070	85.576	8067	1301	7.483
234	VANDNBRG06	34.717	120.550	9126	8311	9.227
235	VINTHILL51	38.733	77.667	5713	1651	0.706
236	WARREN 26	42.467	83.017	5509	2939	4.618
237	WARWICK 44	41.700	71.383	4568	1198	0.212
238	WASHNGTN11	38.900	77.017	5621	1581	10.074
239	WESTOVER25	42.200	72.550	4599	1415	0.814
240	WGHTPTSN39	39.817	84.050	6038	2699	4.416
241	WHIDBEYI53	48.346	122.651	6178	8553	0.536
242	WHITEMAN29	38.733	93.550	7029	4010	2.566
243	WILDER 16	43.667	116.917	7101	7980	0.266

Figure A-1. Listing of SVIP Locations (Continued)

244	WILLIAMS04	35.250	112.183	8756	6855	0.088
245	WINCHSTR51	39.167	78.200	5686	1779	1.359
246	WINOOSKI50	44.500	73.183	4264	1309	0.157
247	WNTRHR BR23	44.400	68.083	3774	1183	0.297
248	WRTHNG TN39	40.083	83.033	5953	2572	0.159
249	WURTSMTH26	44.451	83.394	5170	3115	1.775
250	YAKIMA 53	46.600	120.500	6533	8606	0.832
251	YORKTON CA	51.267	102.467	5003	6273	0.116

Figure A-1. Listing of SVIP Locations (Continued)

2. Step One. The locations that originate conferences are determined by the amount of SVIP traffic originating from them. That is, the number of busy hour conferences originating from a location is assumed to be proportional to the SVIP traffic offered from that location. This rule is implemented in the computer by ranking the locations according to their erlang traffic, partitioning the erlang scale of traffic into a number of intervals, and assigning quantities of originating conferences to locations in these intervals in a manner which increases with increasing traffic. Specifically, an ordered set of erlang cut points, C_0 C_1, C_2 ... are specified so that a location will be assumed to generate i conferences during a busy hour if its SVIP originating erlang traffic is in the range from C_i to C_{i+1} (C_0=0). The selection of these cut points effectively determines the number of conferences in the set of requirements and is the means by which the volume of requirements is controlled. Increasing one or more cut points will lower the overall number of conferences and vice versa. Figure A-2 shows the result of choosing erlang cut points of C_1=5, C_2=8.

Erlang Interval	Number of Locations	Number of Conferences	Total Conferences
0. to 5.0	215	0	0
5.0 to 8.0	24	1	24
greater than 8.0	12	2	24
Total	251		48

Figure A-2. Distribution of Number of Origination Conferences by Location

In this case, 215 of the 251 locations originate no conferences, while 24 locations each generate one conference and 12 locations two. This results in a total of 48 busy hour conferences being generated. The choice of cut points can be parametrically varied, in effect, to throttle the number of conferences in the data base, all the while keeping the rule that the more SVIP traffic emanating from a location, the greater the number of SVIP conferences being generated.

3. Step Two. For each originating conferencing requirement, it is next necessary to know the number of conferees associated with it. This is accomplished by Monte Carlo sampling from an assumed probability distribution which is input to the program. The assumed distribution could be Normal or any other type and could be modified by actual data. The distribution currently being used is as follows:

Number of Conferees (Excluding Originator)	Probability
1	0.0
2	0.083
3	0.125
4	0.167
5	0.167
6	0.125
7	0.104
8	0.083
9	0.063
10	0.042
11	0.021
12	0.021

First, no probability is assigned to one conferee; this event is equivalent to a two-party call. Second, it was anticipated that the majority of conferences would have somewhere around four to five conferees, so these values show the highest probability. From 6 to 12 conferees the probability decreases regularly. The average, or expected number of conferees, per conference for this distribution is 5.66 (or an expected 271 conferees over the 48 conferences of step one). One Monte Carlo sampling distribution of the 48 conferences according to the number of conferees each conference generated is shown below:

NUMBER OF CONFEREES	NUMBER OF CONFERENCES	
	Observed	Expected
1	0	0.
2	7	4.
3	6	6.
4	5	8.
5	12	8.
6	4	6.
7	5	5.
8	3	4.
9	2	3.
10	4	2.
11	0	1.
12	0	1.
Total	48	48

This distribution can be easily altered, if more specific data becomes available.

4. Step Three. Having determined, for each conference, the number of conferees by sampling from an a priori distribution, it is then necessary to determine the community-of-interest or destination locations for the conferees. A detailed mission analysis of each location could provide this data. However, until such specific information becomes available, it is assumed that the conferees are geographically located proportional to the SVIP originating traffic emanating from the locations. A probability distribution is created in which the probability of selecting a conferee location is equal to the location's fraction of total SVIP erlang traffic. One sample from this distribution is made for each conferee of every conference. In this manner, the locations for the conferees are selected randomly, but in proportion to the amount of SVIP traffic they represent. No attempt is made to prevent multiple occurrences of a conferee location for a single conference, or of a conferee being collocated with the originator. Figure A-3 is a listing of the set of 48 conferences showing on each line the conference number (1 thru 48), location number (1 thru 251) of the originator, the number of conferees, and the locations of the conferees (1 thru 251). The location numbers refer to the ordinal position in the listing of Figure A-1.

Holding time statistics for a SVIP conference are generally unavailable. An expected holding time of 10 minutes, constant for all conferences, is currently being assumed. On the originating side, this would equate to 48/6 = 8 erlangs of traffic, and on the destination (conferee) side 271/6 = 45.17 erlangs which would correspond to 271 independent two-party calls. The effect of these conference requirements upon current CONUS AUTOVON is very much dependent upon the number of conference directors in the network and the routing of calls from (1) originating location to local CD, (2) CD to CD spanning all CD's active for a given conference, and (3) each remote CD to its associated set of conferees. The effect of these routings as well as placement and quantity of conference directors is discussed in the main body of this report.

CONF #	ORIG LOC#	NUM OF CONFEREES	LOCATION NUMBERS OF CONFEREES									
1	10	7	29	84	43	48	183	84	60			
2	21	5	105	88	33	179	102					
3	48	3	148	242	179							
4	55	4	78	79	84	106						
5	60	3	170	49	202							
6	60	8	95	236	179	136	12	104	36	213		
7	68	5	223	104	93	102	86					
8	68	4	104	96	113	148						
9	78	5	179	196	148	98	221					
10	78	3	231	33	78							
11	80	7	102	117	91	185	124	93	176			
12	84	2	98	136								
13	85	2	98	218								
14	88	2	124	72								
15	97	2	67	221								
16	98	6	236	194	40	134	88	53				
17	102	2	12	225								
18	102	3	105	181	102							
19	104	10	61	148	1	206	233	31	60	84	92	115
20	105	7	122	28	48	55	188	78	195			
21	105	6	78	126	181	224	10	29				
22	115	2	218	147								
23	134	5	180	5	201	183	31					
24	134	10	42	245	21	103	213	169	55	233	82	92
25	146	5	88	242	66	68	134					
26	146	4	6	201	190	60						
27	147	4	29	66	134	117						
28	158	3	98	92	183							
29	162	5	223	208	240	85	218					
30	165	7	72	8	62	201	90	167	29			
31	179	5	103	218	201	202	181					
32	179	9	24	68	181	104	15	163	106	218	102	
33	183	4	194	98	84	102						
34	186	9	176	88	201	36	91	60	29	239	81	
35	194	8	108	104	127	92	147	117	179	238		
36	194	5	117	48	141	68	78					
37	195	6	223	104	96	105	103	174				
38	201	5	180	67	202	68	218					
39	202	5	165	43	195	195	158					
40	208	6	80	208	194	30	134	110				
41	208	2	104	218								
42	213	7	43	158	179	162	148	200	158			
43	218	5	80	118	136	238	10					
44	233	5	60	100	195	102	84					
45	234	3	117	102	21							
46	234	10	87	61	88	98	105	84	238	80	233	67
47	238	8	60	160	99	163	134	97	134	234		
48	238	10	92	99	81	85	92	102	104	234	15	195

FIGURE A-3 LISTING OF CONFERENCE REQUIREMENTS

APPENDIX B

CONFERENCE DIRECTOR PERFORMANCE AND SIZING MODEL

1. INTRODUCTION

In this study we have assumed that each conference director had to be collocated with a switch. The switch and the conference director are connected via ports on each equipment. When a conference call requires use of the conference director, it will require the use of more than one port. This number will depend on such things as the number of conferees in the conference, the location of the conferees in the conference and whether this particular switch/conference director pair is in a tandem path for the conference. Thus, calls will arrive at the ports and require a random number of ports, depending on the particular conference. Since we do not allow the buffering of calls, the performance of the calls requesting use of these ports can be considered as a loss queueing system where customers (calls) may require more than one server (ports).

To be capable of quickly and efficiently determining the number of ports required to ensure a given level of blocking, we need to develop a mathematical performance model for this system. It has not been until recently that queueing systems where customers require the use of more than one server have been investigated [1]-[5]. In those papers, queueing of customers was allowed. In a different context reference [6] solved the same problem we are discussing here. We did not discover their work until we had independently developed our analysis, and as such we discuss our development, which is tailored to our application.

In section 2 of this appendix, we develop a mathematical model that can be used to predict the performance of the system. Also discussed in that section is a methodology for the sizing of the ports so as to ensure a desired grade of service. Section 3 contains an extensive numerical investigation of the system and possible sensitivities.

2. MATHEMATICAL PERFORMANCE MODEL

Let S be the number of ports connecting the conference director and the switch. We assume the arrival process of calls to the ports is Poisson with parameter λ; and that Q_n, $n=1,2,\ldots N$, is the probability the call requests n ports. Without loss in generality we further assume $N \leq S$. If an arriving call requesting the use of n ports does not find n ports free it is dismissed from the system. Let us assume the holding time of a call requesting n ports is exponientially distributed with mean μ_n^{-1}. We assume that the call maintains control of the n ports for its entire holding time, at the conclusion of which time it releases all of the ports.

Let C_n be the steady state number of conference calls, who requested n ports, in the system and define

$$P_{i_1, i_2, \ldots, i_N} = \Pr\{C_1 = i_1,\ C_2 = i_2, \ldots,\ C_N = i_N\}, \qquad (B.1)$$

with PL_n being the probability a call requesting n ports is lost. It is intuitively obvious that

$$PL_1 \le PL_2 \le \ldots \le PL_N \tag{B.2}$$

since a call requesting n ports is not blocked as much as a call requesting (n+1) ports and we do not allow any preempting.

Let

$$S^* = \max\{r \mid r \le S - \sum_{t=1}^{N} i_t \text{ and } r=0,1,\ldots,N\} \tag{B.3}$$

then the steady state equations for $P_{i_1,i_2,\ldots,i_N}$ are

$$\left[\sum_{n=1}^{S^*} \lambda Q_n + \sum_{n=1}^{N} i_n \mu_n\right] P_{i_1,i_2,\ldots,i_N} = \sum_{n=1}^{N} \lambda Q_n P_{i_1,\ldots,i_n-1,\ldots,i_N} + \sum_{n=1}^{S^*} (i_n+1)\mu_n P_{i_1,\ldots,i_n+1,\ldots,i_N} \tag{B.4}$$

when $\sum_{r=1}^{N} ri_r \leq S$; $P_{i_1,i_2,\ldots,i_N} = 0$ if there exist an $i_r < 0$ or if

$$\sum_{r=1}^{N} ri_r > S.$$

It is straightforward to show that if $\rho_n = \lambda Q_n/\mu_n$ for $n=1,2,\ldots,N$; the solution to these equations is

$$P_{i_1,i_2\ldots,i_N} = \frac{\rho_1^{i_1}}{i_1!} \frac{\rho_2^{i_2}}{i_2!} \cdots \frac{\rho_N^{i_N}}{i_N!} P_{0,0,\ldots 0}, \tag{B.5}$$

where $P_{0,0,\ldots.,0}$ is found by the normalizing condition to be

$$P_{0,0,\ldots,0}^{-1} = \sum_{r_1=0}^{S} \sum_{r_2=0}^{[\frac{S-r_1}{2}]} \cdots \sum_{r_N=0}^{[\frac{S-r_1 \ldots-(N-1)r_{N-1}}{N}]} \prod_{r=1}^{N} \frac{\rho_r^{i_r}}{i_r!} \tag{B.6}$$

where $[x]$ is the greatest integer $\leq x$. So the problem rests on being able to evaluate the sum given in equation (B.6). Arthurs and Kaufman obtained the same solution for their problem and also showed that the results hold for general holding times.

The following iterative scheme can be used to quickly evaluate this sum. For $j=0,1,2,\ldots S$ define

$$f(N,j) = \sum_{r=0}^{[j/N]} \frac{\rho_N^{\,r}}{r!}; \tag{B.7}$$

and for $i=N-1, N-2, \ldots, 2, 1$ and $j = 0, 1, \ldots, S$ define

$$f(i,j) = \sum_{r=0}^{[j/i]} \frac{\rho_i^{\,r}}{r!}\, f(i+1,j-ir). \tag{B.8}$$

It is interesting to reflect on what $f(i,j)$ represents and to draw some parallels between this recursion scheme and those that appear in Dynamic Programming. The quantity $f(i,j)$ can be considered as one has j ports to be distributed among the call requesting i, i+1, ..., N ports. In [6] they used the same scheme to determine $P_{0,0,\ldots,0}$; it is similar to the one Buzen [7] used to compute the normalizing constant in the context of network of queues. In that context Kobayashi [8] gives a summary of the methods that have been used to efficiently determine these constants.

The required sum is given by f(1,S), so

$$P_{0,0,\ldots,0} = \frac{1}{f(1,S)} \tag{B.9}$$

We note in evaluating equations (B.7) and (B.8) only two vectors of length S+1 have to be stored in the computer.

It turns out that some additional information is directly contained in f(1,j) for j = 0, 1, ..., S. Let X present the steady state number of busy servers; then with a little thought one sees for j = 0, 1, ..., S

$$\Pr\{X=j\} = \frac{f(1,j)-f(1,j-1)}{f(1,S)} \tag{B.10}$$

where $f(1,-1) = 0$. Using equation (B.10) the loss probabilities PL can be quickly found, for $n = 1, 2, \ldots, N$

$$PL_n = \sum_{r=S+1-n}^{S} Pr\{X=r\} = PL_{n-1} + Pr\{X=S+1-n\} \qquad \text{(B.11)}$$

where $PL_0 = 0$. From equation (B.11) one directly sees the inequality relationship given by equation (B.2). Finally, the overall average loss probability, PL, is

$$PL = \sum_{n=1}^{N} \rho_n PL_n \Big/ \rho \qquad \text{(B.12)}$$

where $\rho = \sum_{n=1}^{N} \rho_n$.

Although equation (B.11) gives us the most desirable measure of performance for the system, the iteration procedure described via equations (B.7) and (B.8) can also be used to give other system performance characteristics. Let $F(T,\theta_n)$ the corresponding value of $f(1,T)$ when we recursively use equations (B.7) and (B.8) with $S=T$ and $\theta_n = (\rho_1,\rho_2,\ldots,\rho_{n-1},0,\rho_{n+1},\ldots,\rho_N)$. The vector θ is equivalent to a vector of the ρ_i's but with ρ_n set to 0. The probability distribution of the number of calls, who requested n ports, in the system is given by

$$\Pr\{C_n=i\} = \frac{\rho_n^i}{i!} \frac{F(S-ni,\bar{\theta}n)}{f(1,S)} \tag{B.13}$$

for i = 0,1,..., [S/n] and n = 1, 2, ..., N.

The same procedure may be used to evaluate the joint probability of C_m and C_n for any m and n. If $\theta_{m,n}$ is the vector of ρ_i's with $\rho_m=\rho_n=0$ then

$$\Pr\{C_m=i,C_n=j\} = \frac{\rho_m^i}{i!} \frac{\rho_n^j}{j!} \frac{F(S-im-jn,\bar{\theta}_{m,n})}{f(1,S)} \tag{B.14}$$

Using the concept of carried load [9] it is easy to relate PL and the expected value of C_n, $E\{C_n\}$. In steady state, we must have the expected number of calls requesting n ports in the system equal to the offered load for that class times the probability it is not lost; so for n = 1, 2, ..., N we have

$$E\{C_n\} = \rho_n(1-PL_n). \tag{B.15}$$

Before discussing how one uses these results to quickly and efficiently determine the required number of ports necessary to ensure a desired grade of service to be met, a method of computing the probability distribution of the number of conferences in the system is discussed. Let C represent the steady state number of conferences in the system; then it is easy to show that for $i \leq [S/N]$

$$\Pr\{C=i\} = \frac{\rho^i/i!}{f(1,S)}; \tag{B.16}$$

but for $i \geq [S/N]+1$ the problem is much more difficult. For these cases we use another iterative scheme to generate the desired results. Define for k = 0, 1, ..., S

$$h_k(N,j) = \begin{cases} \dfrac{\rho_N^j}{j!} & :j=0,1,\ldots,[\frac{k}{N}] \\ 0 & :j=[\frac{k}{N}]+1,\ldots,k, \end{cases} \tag{B.17}$$

and again the backward relation for n = N-1, N-2, ..., 2, 1

$$h_k(n,j) = \begin{cases} \sum_{r=0}^{j} \frac{\rho_n^r}{r!} h_{k-rn}(n+1,j-r) & : j=0,1,\ldots,[\frac{k}{n}] \\ 0 & : j=[\frac{k}{n}]+1,\ldots,k \end{cases} \quad (B.18)$$

For $i = [S/N]+1, \ldots, S$ we have

$$\Pr\{C=i\} = \frac{h_S(1,i)}{f(1,S)} . \quad (B.19)$$

It is interesting to consider the amount of computation and storage required to compute the probability distribution of C and X. For both, one has to store two vectors; but for X the length of the vector is S+1, whereas it is (S+1)+(S+2)/2 for C using the storage mapping

$$h_k(n,j) \rightarrow H(n,\ell) \quad (B.20)$$

where $\ell = k(k+1)/2 + j$. So the computational and storage requirements to produce the probability distribution of C are greater than for X.

There are several other relations between C and X which should be presented. First the expected number of conferences in the system is

$$E\{C\} = \sum_{n=1}^{N} E\{C_n\}$$

$$= \sum_{n=1}^{N} \rho_n (1-PL_n). \tag{B.21}$$

Using the notion of carried load we have

$$E\{x\} = \sum_{n=1}^{N} n\rho_n (1-PL_n). \tag{B.22}$$

We close this section with a discussion of the sizing routine used in this study. Supposing PL* is the desired average loss probability, we want to know what value of S will achieve this probability. For the standard Erlang Loss system, the problem is straightforward because the loss probability with S+1 ports is simply expressed as a function of the loss probability with S ports, [9]. So one can iteratively increase S until the desired grade of service is achieved. Since the loss probability is monotonically decreasing in S no other checks need be done.

In the context of the conference director port sizing, the problem is not so simple. First, as one will see in the next section, PL is not monotonically decreasing in S for a fixed load. Secondly, there is no simple way of determining PL for S+1 from PL with S ports. Our sizing method is based on the following observation from the numerical examples we have considered. If $E(\rho, S)$ is Erlang's Loss Formula, then for ρ_n (n=1, 2, ..., N) fixed

$$E(\rho,[S/\sum_{n=1}^{N} rQr]) \to PL \tag{B.23}$$

as S gets large. Since PL is getting small as S is increased and the values of PL* are usually less than .1, the sizing procedure first determines the required number of ports, say $\bar{S}$, such that

$$E(\rho,[\bar{S}/\sum_{n=1}^{N} nQn]) \leq PL^*. \tag{B.24}$$

Once $\bar{S}$ has been found, the average loss probability is computed using S and ρ_n's. If it is greater than PL*, $\bar{S}$ is decreased until the average loss probability is greater than PL*, at which time $\bar{S}$ is reset to its previous value. What this procedure does is allow one to determine the number of ports without having to evaluate the average loss probability for all values of S less than or equal to S.

3. NUMERICAL ANALYSIS AND SYSTEM PERFORMANCE

In this section we consider some numerical examples using the results of section 2. In general the behavior of this system is very interesting and sometimes extremely sensitive to slight changes in the parameters under consideration.

Figure B-1 shows how radical the behavior of this system can be. As a function of the number of ports, the loss probabilities PL_1, PL_9, and PL are shown. One sees that their behavior is cyclic and that PL_1 is not monotonically decreasing in S. The reason for this strange behavior is simply explained; when the number of ports is less than nine no calls requiring nine ports are accepted into the system. So as S is increased from 1 to 8 PL_1 decreases monotonically from 1 at S = 0 to basically 0 at S = 8. When S = 9, those calls requiring nine ports are allowed into the system and occupy nine ports; thus for periods of time the system has no ports available for the calls requesting one port. This causes PL_1 to jump from 0 at S = 8 to around .78 at S = 9. When S is increased from 9 to 17, only one call requesting nine ports is allowed in the system at a time and PL_1 starts decreasing again. When S = 18, two to nine port calls could be in the system and the cyclic behavior begins all over. The overall behavior is cyclic in the number of ports; with cyclic length equal N and an overall downward drift in the values of loss probabilities.

The five cases shown in Figure B-2 represent the situation where all parameters are held constant but the variance of the offered load is monotonically increasing in Figures B-2.A to B-2.F. Usually, in queueing systems when the variance of one of the underlying random variables is increased, the measures of performance also increase. In these figures, we see the opposite happening; as the variance of the offered load is increasing the average loss probability is decreasing. The basic reason stems from the fact that as the variance of the offered load is increased the variance in the number of requests for ports is increased and the system will be better utilized because requests for a specific number of vacant ports is more likely to occur.

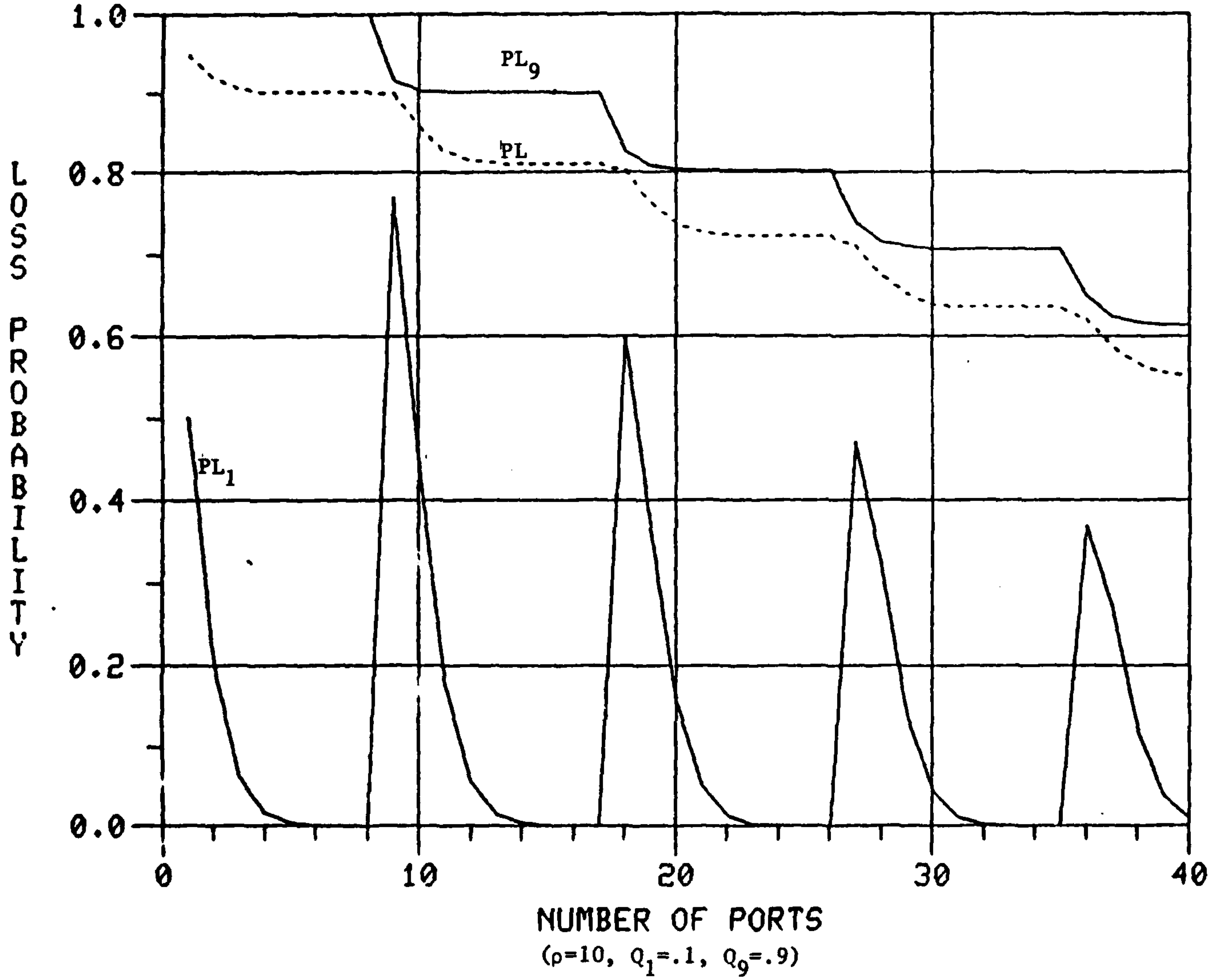


Figure B-1. Cyclic Behavior of Loss Probabilities

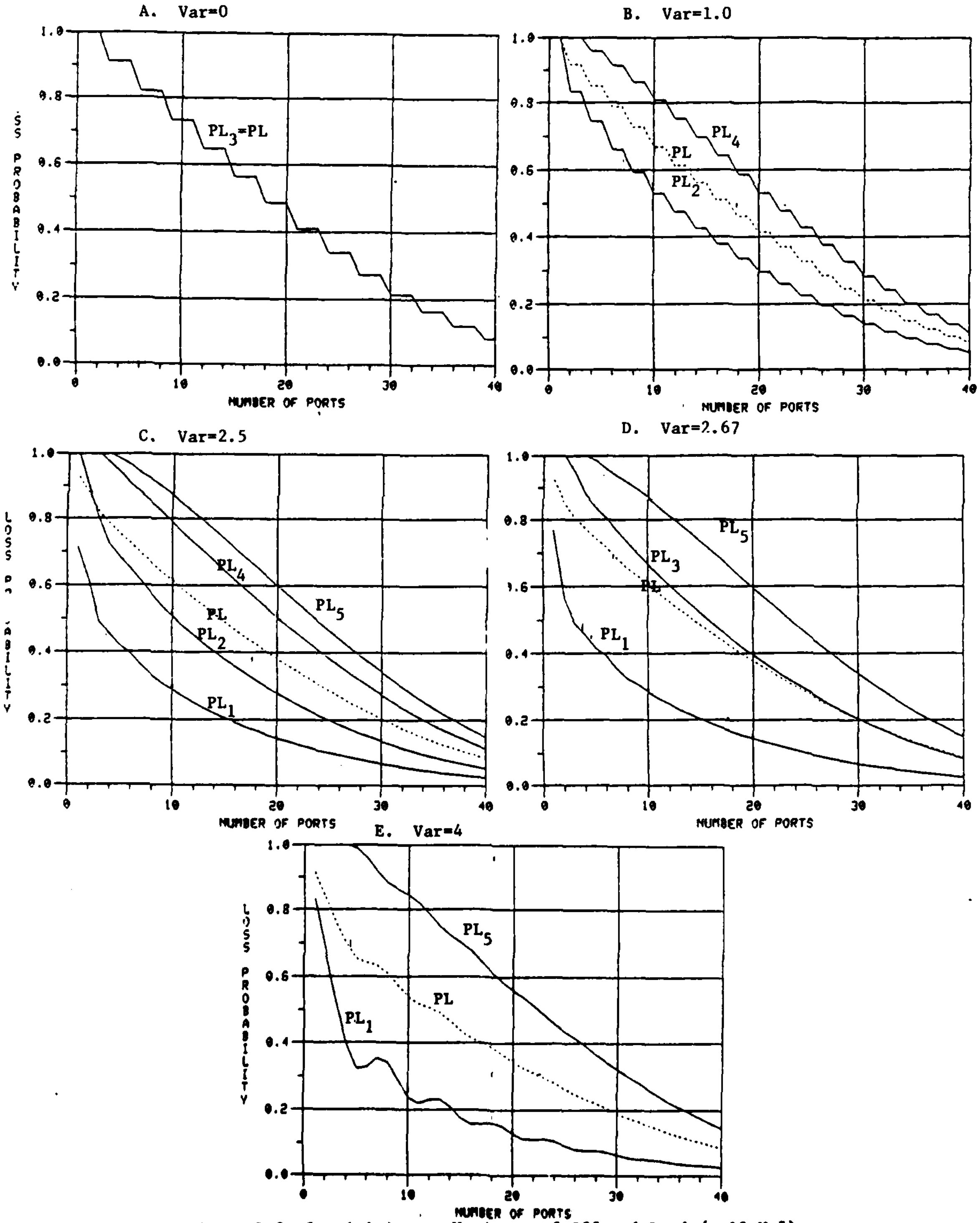


Figure B-2. Sensitivity to Variance of Offered Load (ρ=10,N=5)

In Figure B-3, the case is considered when there are no requests for one port. In that figure, one has the radical behavior of PL_n for a small value of S. This behavior is similar to the one we saw in Figure B-1 but not as pronounced, because we do not have the wide differences in the number of ports requested. Another result we wish to demonstrate via this figure is that $PL_2 = PL_3$ when $S = 3$, $PL_3 = PL_4$ when $S = 4$ and $PL_4 = PL_5$ when $S = 5$. The reason we have equality at these points is that we only can have one type of call in the system at a given time, not both; and since $Q_1 = 0$ a request for n ports is the same as a request for n+1 ports. Once S>N there is more interaction between the requests for ports and the strange behavior disappears.

The interaction among the number of calls requesting 1, 2, and 3 ports is shown in Figure B-4. The covariance of each pair of possible requests is plotted as a function of S. One immediately sees that there is a high negative correlation of the number of calls requesting a different number of ports. In this example there is a high correlation between calls requesting two and three ports. Basically all these curves are convex in nature. This stems from the fact that when the number of ports is small, the loss probabilities are high, and there are not many customers in the system; i.e., covariance is small. As the number of ports is increased, more and more calls from each class are accepted and the correlation becomes greater until the number of ports is large enough to ensure that the calls begin to act independently of each other. Again we see the radical performance for small S.

The final figure (Figure B-5) gives a family of curves for $Pr\{C_n = i\}$, n = 1, 2, 3 and $Pr\{C = i\}$. This figure does not show any of the radical behavior that we have seen in the previous figures. As expected, the variance in the number of calls requesting one port is greater than those requesting two; which is greater than those requesting three. Furthermore, the variance of the number of conferences present is not equal to the sum of the variance of the conference requesting a particular number of ports because of the dependencies among the underlying variables.

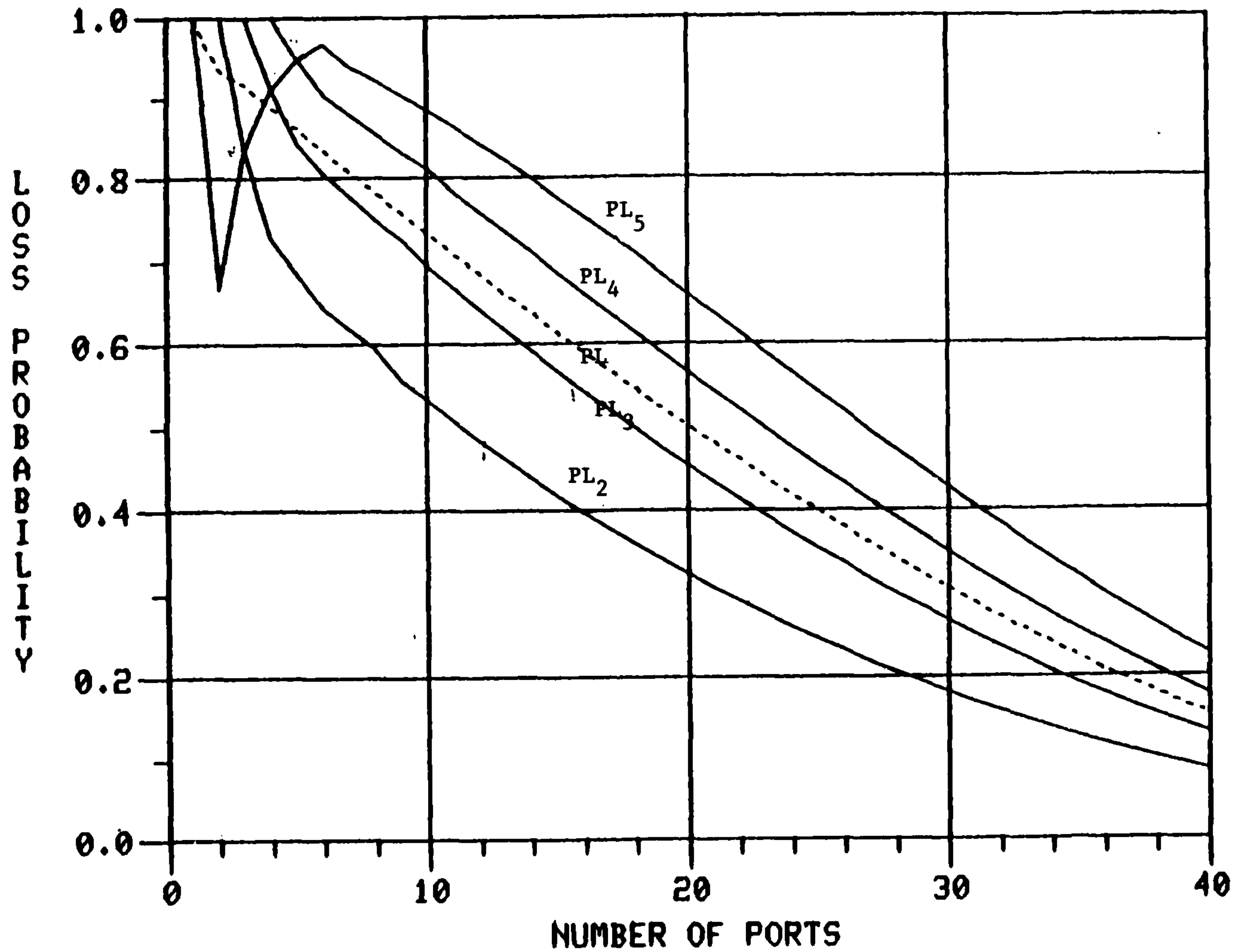


Figure B-3. Behavior of System When $Q_1=0$ ($\rho=10$, $Q_2=Q_5=.2, Q_3=Q_4=.3$)

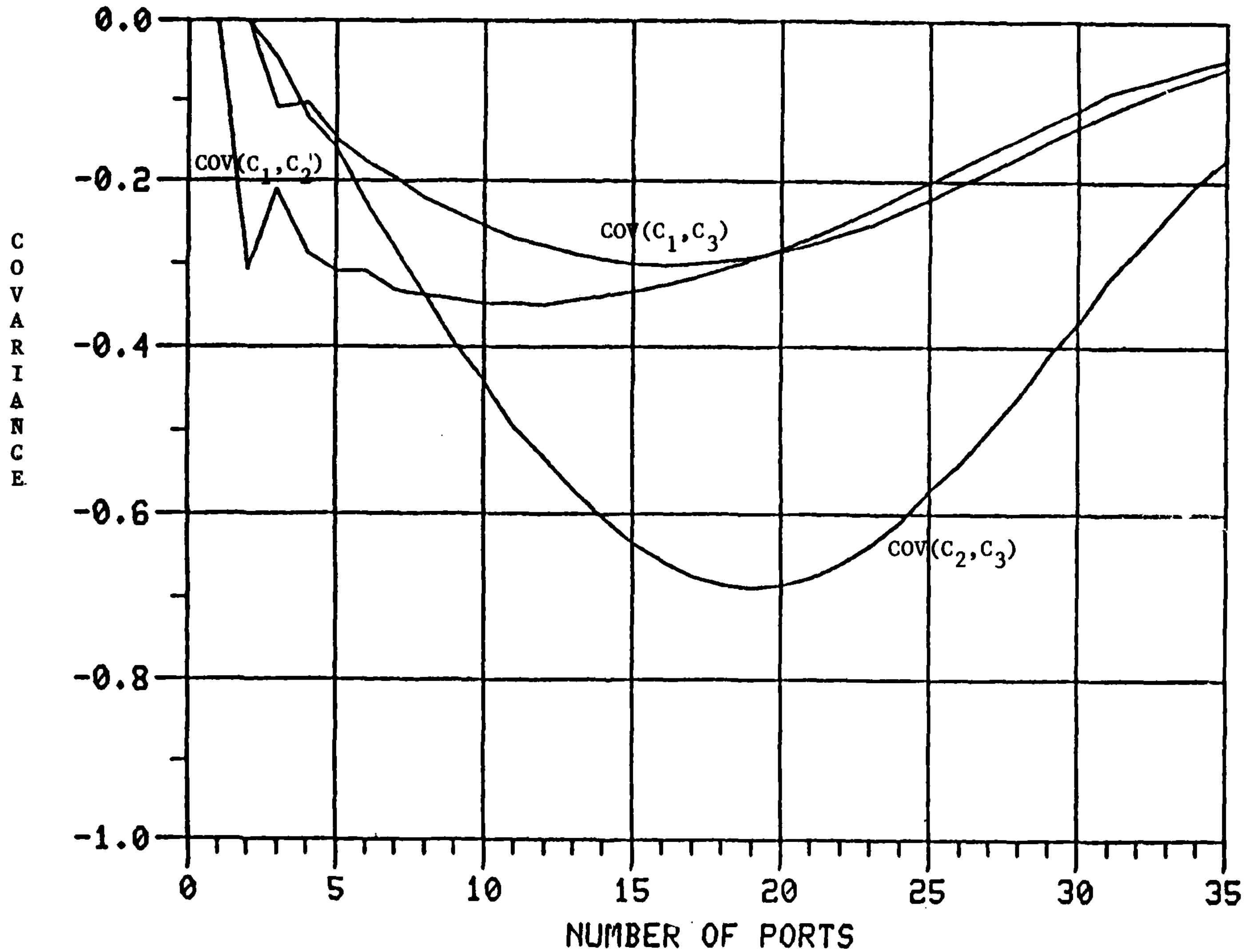


Figure B-4. Covariance of Request for Ports
(ρ=10, Q_1=.3, Q_2=.4, Q_3=.3)

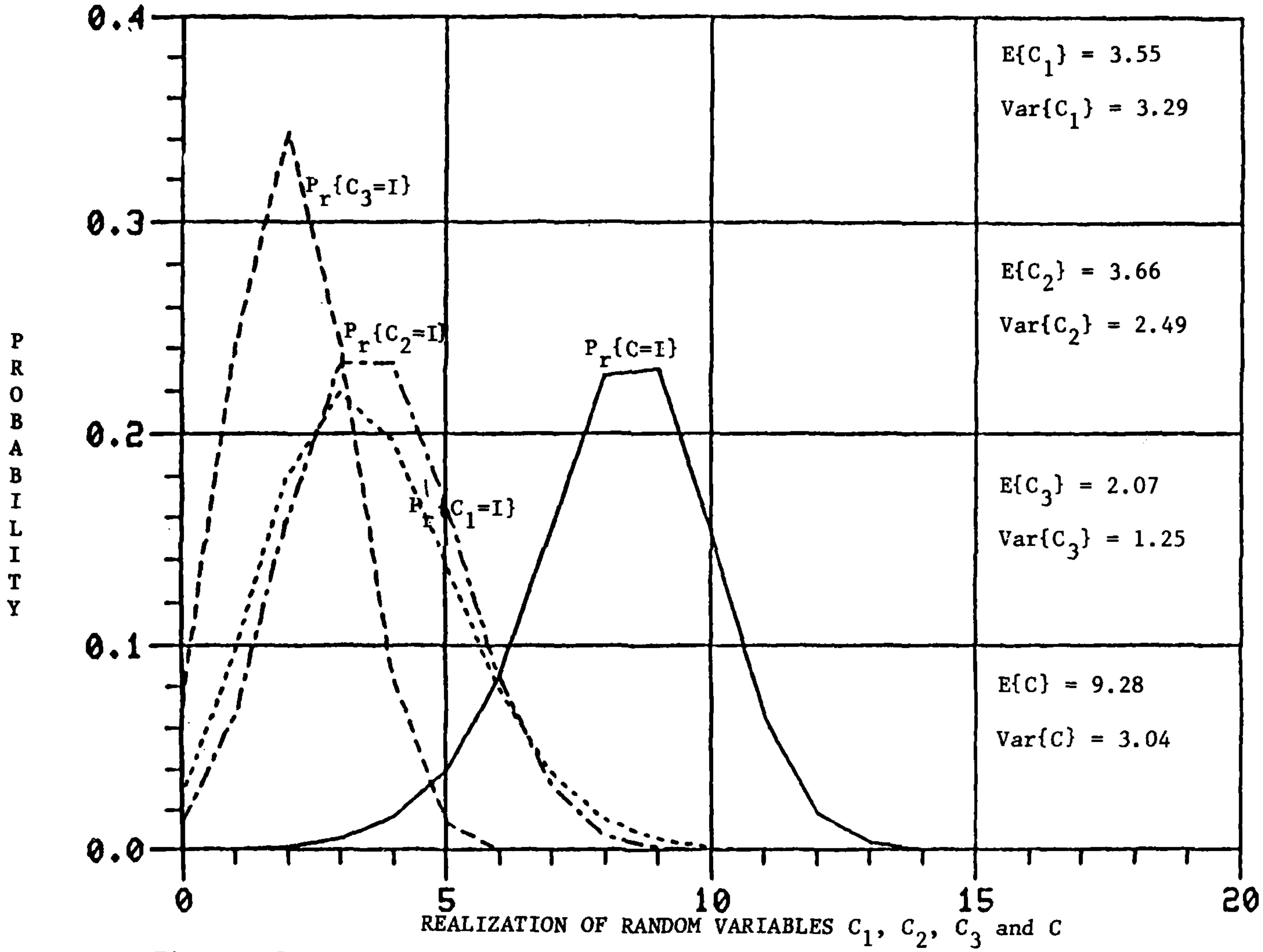


Figure B-5. Probability Distribution of the Number of Calls in System by Class ($\rho=15$, $Q_1=.3$, $Q_2=.4$, $Q_3=.3$, $S=20$)

REFERENCES

[1] L. Green, "Queues Which Allow a Random Number of Servers per Customer," Ph.D. Dissertation, Yale University, 1978.

[2] L. Green, "A Queueing System in Which Customers Require a Random Number of Servers, " Operations Research, 28, 1980.

[3] L. Green, "Comparing Operating Characteristics of Queues in Which Customers Require a Random Number of Servers," Working Paper, Graduate School of Business, Columbia University, New York.

[4] L. Green, "A Queueing System with Auxiliary Servers." Research Working Paper # 334A, Graduate School of Business, Columbia University, New York, June, 1980.

[5] S. Kim "M/M/S Queueing System Where Customers Demand Multiple Server Use," Ph.D. Dissertation, Southern Methodist University, 1979.

[6] E. Arthurs and J. S. Kaufman, "Sizing a Message Store Subject to Blocking Criteria," 4th International Symposium on Modelling and Performance Evolution of Computer Systems, Feb 6-8, 1979, Vienna, Austria.

[7] J. P. Buzen, "Computational Algorithms for Closed Queueing Networks with Experimential Servers." Communications of the ACM, Vol. 16, No. 9, 1973.

[8] H. Kobayashi, Modeling and Analysis: An Introduction to System Performance Evolution Methodology, Addison - Wesley, Reading, Mass., 1978.

[9] R. B. Cooper, Introduction to Queueing Theory, Macmillan, New York, 1972.

DISTRIBUTION LIST

R100 - 2
R102/R103/R103R - 1
R102M - 1
R102T - 9 (8 for stock)
R104 - 1
R110 - 1
R123 - 1 (Library)
R124A - 1 (for Archives)

R200 - 1
R300 - 1
R400 - 1
R500 - 1
R700 - 1
R800 - 1
NCS-TS - 1
101A - 1
312 - 1

R102T - 12

DCA-EUR - 2 (Defense Communications Agency European Area
ATTN: Technical Director
APO New York 09131)

DCA-PAC - 3 (Commander
Defense Communications Agency Pacific Area
Wheeler AFB, HI 96854)

1 (Commander
DCA-Southwest Pacific Region
APO San Francisco 96274)

1 (Commander
DCA-Northwest Pacific Region
APO San Francisco 96328)

1 (Chief
DCA-Korea Field Office
APO San Francisco 96301)

1 (Chief
DCA-Okinawa Field Office
FPO Seattle 98773)

1 (Chief
DCA-Guam Field Office
Box 141, NAVCAMS WESTPAC
FPO San Francisco 96630)

1 (U.S. Naval Short Electronic Engineering
Activity Pacific
Box 130, ATTN: Code 420
Pearl Harbor, HI 96860)

1 (1843 EE Squadron
ATTN: EIEXM
Hickam AFB, HI 96853)

DISTRIBUTION (Continued)

DCA FO ITALY - 1 (DCA Field Office Italy, Box 166
AFSOUTH (NATO), FPO New York 90524)

USDCFO - 1 (Chief
USDCFO/US NATO
APO New York 90667)

DATE
FILME
6–8